THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATION – 2023

FIRST YEAR BACHELOR OF SCIENCE IN APPLIED MATHEMATICS

AM 125 – LINEAR ALGEBRA I

TIME ALLOWED: 3 HOURS

INSTRUCTIONS FOR CANDIDATES:

1. You have 10 minutes to read through this paper. You must **NOT** begin writing during this time.
2. There are five (5) questions. Answer **ALL** questions.
3. Write all answers in the answer booklet(s) provided.
4. All workings should be shown clearly in the answer booklet(s).
5. Start each question on a new page and clearly write its question number at the top of the page.
6. Calculators are allowed in the examination room.
7. Mobile phones **must** be switched off during the examination period.
8. Make sure that your **name**, **surname** and **ID number** are clearly written on the front of the examination answer booklet(s).
9. Required formulas are provide at the end of the question paper.

MARKING SCHEME

Questions carry marks as indicated. Total marks: 80

Question 1: $[(\mathbf{1+1+1+1+1+1+1+1+1+1}) = \mathbf{11\ marks}]$

Write True (T) or False (F) for each of these statements:

(i) Two matrices are said to be equal only if the number of rows in both matrices are equal.

(ii) If all elements except diagonal elements of a square matrix are zero, then the matrix is said to be a diagonal matrix.

(iii) The addition or subtraction of two or more matrices is possible only when they are of the same order.

(iv) For the multiplication of two matrices A and B, the number of columns of matrix A and the number of rows of matrix B should not be equal.

(v) Matrix multiplication is commutative.

(vi) Associated with each eigenvalues are unique eigenvectors.

(vii) If $A = \begin{bmatrix} 4 & 5 \\ -2 & 3 \end{bmatrix}$, then $(A^T)^T = \begin{bmatrix} 4 & 5 \\ -2 & 3 \end{bmatrix}$.

(viii) Adjoint of a square matrix is the transpose of the matrix of the co-factors of a given matrix.

(ix) The system of linear equations $\begin{cases} x + 2y = 11 \\ -2x - 4y = 22 \end{cases}$ has infinitely many solutions.

(x) Determinant of a triangular matrix is equal to the sum of the main diagonal.

(xi) If a square matrix A has a row (or column) of zeros, then $|A| = 1$.

Question 2: $[(\mathbf{4+1}) + (\mathbf{4+1}) + (\mathbf{4+1}) + (\mathbf{4+1}) = \mathbf{20}\boldsymbol{marks}]$

(a) What is an identity matrix? Give an example of a 2x2 identity matrix.

(b) Define the following and give an example for each:

(i) Square matrix

(ii) Diagonal Matrix

(iii) Triangular Matrix.

Question 3: $[\mathbf{8 + 10} = \mathbf{18}\boldsymbol{marks}]$

(a) The $3x3$ Matrix A is defined in terms of the scalar constant k by A = $\begin{bmatrix} 2 & -1 & 3 \\ k & 2 & 4 \\ k-2 & 3 & k+7 \end{bmatrix}$.

Given that $|A| = 8$, find the possible values of k.

(b) Determine the eigenvalues and eigenvectors for the equations $Ax = \lambda x$ where $A = \begin{bmatrix} 1 & 8 \\ 8 & -11 \end{bmatrix}$.

Question 4: $[1 + 4 + 2 + 3 + 7 = 17\ marks]$

Given the system $\begin{cases} x + 3y + 2z = 14 \\ 2x + y + z = 7 \\ 3x + 2y - z = 7 \end{cases}$

(i) Form the augmented matrix.
(ii) Apply elementary row operations to reduce to its triangular matrix form.
(iii) Using the answer of part (ii), determine the rank of the matrix.
(iv) Using the result of part (ii), solve the system of equations.
(v) Prove the solution obtain in part (iv) using Cramer's Rule.

Question 5: $[8 + 6 = 14\ marks]$

Given a $3x3$ matrix A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$:

(a) Find the inverse of matrix A using the formula $\boldsymbol{A^{-1} = \frac{1}{|A|}adj(A)}$.

(b) Hence or otherwise, solve these system of linear equations

$$\begin{cases} x + 2y + z = 1 \\ 2x + 3y + z = 4 \\ 3x + 4y + 2z = 4 \end{cases}$$

END OF EXAMINATION

Formula Sheet

Name of Rule	Formula
Cramer's Rule	$x_1 = \frac{Dx_1}{D}, x_2 = \frac{Dx_2}{D}, \ldots, x_n = \frac{Dx_n}{D}$
Inverse Matrix	$A^{-1} = \frac{1}{\lvert A \rvert} adj(A)$
Inverse Method	$X = A^{-1}B$
Characteristic equation	$\lvert A - \lambda I \rvert = 0$ $\lvert A - \lambda I \rvert \boldsymbol{x} = 0$