THE PAPUA NEW UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS, 2023

SECOND YEAR APPLIED MATHEMATICS

AM226 NUMERICAL METHODS I

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS TO CANDIDATES

1. You have 10 minutes to read this paper. You **must not** begin writing during this time.
2. There are **five (5)** questions in this examination paper. Answer **all** questions.
3. Write all answers in the answer booklet provided.
4. All working should be shown clearly on the answer booklets.
5. All rough work should be **crossed out** leaving the final answer for clarity.
6. Start each question on a new page and clearly write its question number at the top of the page.
7. Calculators are allowed in the examination room.
8. Write your name and id number clearly on the examination booklets.
9. Mobile phones and other recording devices must be **switched off** during the examination period.
10. The last page is the formula and information sheet.

MARKING SCHEME

Marks are indicated at the beginning of each question. The total mark is **100 marks.**

QUESTION 1 [10 + 3 + 7 = 20 Marks]

Let $f(x) = e^{x}\cos x$. Answer the following questions accordingly. (Leave the final answers correct to 4 decimal places where necessary.)

(a) Find the third Taylor polynomial $P_3(x)$ for $f(x)$ about $a = 0$.

(b) Apply $P_3(0.1)$ to approximate $f(0.1)$.

(c) Find the upper bound error $R_3(0.1)$ and compare it the actual error $E_3(0.1)$ using the error formula.

QUESTION 2 [5 + 3 + 5 + 7 = 20 Marks]

(a) Study the given matrices and answer the questions that follow.

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & -3 \\ -2 & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & -5 & 1 \\ 1 & 3 & 0 \\ 3 & 4 & -1 \end{bmatrix}, \quad \mathbf{C} = \frac{1}{27}\begin{bmatrix} 2 & -1 & -11 \\ 5 & 11 & 13 \\ 4 & -2 & 5 \end{bmatrix}$$

(i) Find **AB**.

(ii) Find the determinant of **B**.

(iii) Show that **C** is the inverse of **A**.

(b) Find the approximate solutions to the following linear system of equations using the Jacobi method with $X^{(0)} = 0$. Only the first two (2) iterations are required. (Round off the final answers correct to 4 decimal places.)

$$\begin{cases} 5x - 2y + w = 5 \\ x + 9y + z - 3w = -9 \\ -2x + 3y + 10z + 2w = 10 \\ 3x - z + 8w = 8 \end{cases}$$

QUESTION 3 [8 + 5 + 7 + 5 = 25 Marks]

(a) Apply the indicated numerical integration methods to evaluate the given integral.(Leave the final answer correct to 4 decimal places.)

$$\int_0^2 2xe^{-x}\,dx; \quad n = 4.$$

(i) Use the Trapezoidal method to evaluate the integral.

(ii) Use the Simpson's $\frac{1}{3}$ rule to evaluate the integral.

(iii) Use the Gauss Quadrature 2 point rule to evaluate the integral.

(b) Find the error bound for part (a) (i) above.

QUESTION 4 **[10 + 10 = 20 Marks]**

(a) Solve the following initial value problem for y when $x = 2$ using the Euler method with $h = 1$. (Leave the final answer correct to 4 decimal places.)

$$\frac{dy}{dx} = 2xy - e^{-2x}; \quad y(0) = -1$$

(b) Repeat part (a) using the Runge – Kutta 4th order method with $h = 2$. (Leave the final answer correct to 4 decimal places.)

QUESTION 5 **[10 + 5 = 15 Marks]**

(a) Find the eigenvalues and their corresponding eigenvectors for the given matrix.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$$

(b) For the 3 x 3 matrix $\mathbf{A} = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 3 & -2 \\ 1 & -2 & 3 \end{pmatrix}$, its eigenvalues are: $\lambda_1 = 2$, $\lambda_2 = \frac{7+\sqrt{33}}{2}$ & $\lambda_3 = \frac{7-\sqrt{33}}{2}$. Find the eigenvector corresponding to $\lambda_1 = 2$.

END OF EXAMINATION

FORMULA/INFORMATION SHEET

1. $P_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots + \frac{f^n(0)x^n}{n!}$

2. $P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots + \frac{f^n(a)(x-a)^n}{n!}$

3. Error in $T_n \le \frac{M(b-a)^3}{12n^2}$; $M = Max\ of\ |f''(x)|$ over $[a,b]$.

4. Error in $S_n \le \frac{M(b-a)^5}{180n^4}$; $M = Max\ of\ |f^{(4)}(x)|$ over $[a,b]$.

5. $\int_a^b f(x)dx = \frac{h}{2}[f_0 + 2f_1 + 2f_2 + \ldots + 2f_{n-1} + f_n]$; $h = \frac{b-a}{n}$

6. $\int_a^b f(x)dx = \frac{h}{3}[f_0 + 4f_1 + 2f_2 + \ldots + 4f_{n-1} + f_n]$; $h = \frac{b-a}{n}$

7. $\int_a^b f(x)dx = \frac{3h}{8}[f_0 + 3f_1 + 3f_2 + 2f_3 + 3f4 + 3f5 + 2f6 + \ldots + f_n]$; $h = \frac{b-a}{n}$

8. $\frac{dy}{dx} = f(x,y); \quad y(x_o) = y_0.$

   (a) $y_{n+1} = y_n + hf(x_n, y_n)$, h will be given.

   (b) $y_{n+1} = y_n + \frac{h}{6}[k_1 + 2k_2 + 2k_3 + k_4]$, h will be given. Here,

   $k_1 = f(x_n, y_n)$;
   $k_2 = f(x_n + 0.5h, y_n + 0.5hk_1)$;
   $k_3 = f(x_n + 0.5h, y_n + 0.5hk_2)$;
   $k_4 = f(x_n + h, y_n + hk_3)$.

9. $\mathbf{Ax} = \lambda\mathbf{x}$
   $(\mathbf{A} - \mathbf{I}\lambda)\mathbf{x} = \mathbf{0}$
   $|\mathbf{A} - \mathbf{I}\lambda| = 0$

10. $R_n(x) = \frac{f^{(n+1)}(\zeta(x))}{(n+1)!}(x-a)^{n+1}$

11. $\int_a^b f(x)dx \approx c_1 f(x_1) + c_2 f(x_2)$ where,

   $c_1 = \frac{b-a}{2}$, $c_2 = \frac{b-a}{2}$, $x_1 = \frac{b-a}{2}\left(\frac{-1}{\sqrt{3}}\right) + \frac{b+a}{2}$, $x_2 = \frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}$