



**THE PAPUA NEW GUINEA
UNIVERSITY OF TECHNOLOGY**

**DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
FIRST SEMESTER EXAMINATIONS 2022**

FIRST YEAR APPLIED MATHEMATICS

AM111 - FOUNDATION MATHEMATICS

TIME ALLOWED – 3 HOURS

INFORMATION FOR CANDIDATES:

1. Write your name, student number, and program of study clearly on the front page of your answer booklet. Do it **now**.
2. You have 10 minutes to read this examination paper. During this time you must **NOT** write **inside** your answer booklet. You can make notes on the examination paper.
3. A scientific calculator is permitted, though **you do not have to use one**. Other electronic devices are not permitted. Notes and headphones are not permitted.
4. At the conclusion of the examination you must **immediately** put your pens down. You are **NOT** permitted to write inside your answer booklet after the "end of examination" announcement.
5. You can answer the questions in any order. Start each question on a new page. After you have finished the exam, indicate the order in which you answered questions in the left column of the marks box on the cover of the answer booklet.
6. There are 6 questions. **You should attempt only 5**. If you attempt all 6 only the first 5 will be marked.

MARKING SCHEME:

Each question is worth 20 marks. The marks breakdown for question parts are indicated at the top of each question.

QUESTION 1 (5 + 5 + 4 + 6 = 20 marks)

- (a) Classify the following real numbers as natural, integer, rational, irrational or decimal. (Some may belong to several categories – select the one that best describes the number).
- (i) 54 (ii) 5.4 (iii) -21 (iv) π (v) $12/7$
- (b) The prime numbers start 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,...
- (i) How would you check if 843 is prime, using an efficient check.
- (ii) Natural numbers that are NOT prime have another name. What is that name?
- (iii) A number of the type in (ii) is 84. Write this as a product of primes.
- (c) Under the operations multiply and add, our number systems are (i) closed, (ii) commutative, (iii) associative and (iv) distributive. Illustrate the meaning of each with an example using natural numbers.
- (d) Without more information, the decimal number 53.74 would usually be considered in to be a measurement, rather than being an exact number.
- (i) What is the difference between being a 'measurement' and being 'exact'?
- (ii) What are the largest and smallest measurements that would be rounded to 53.74 correct to 4 significant places.
- (iii) As a rule of thumb – when do we normally apply significant place accuracy, and when do we apply decimal place accuracy?

QUESTION 2 (3 + 5 + 3 + 9 = 20 marks)

- (a) Consider the decimal (base 10) number 104.032. What is the 'place value' of each digit?
- (b) To write numbers in base 12 we require to more digits – say A and B. Using this, the first 12 non-negative numbers in this base would be 0,1,2,3,4,5,6,7,8,9,A,B.
- (i) Continue this sequence by writing down (in base 12) the next 30 natural numbers in **base 12**. [You can use ... where you think the transition from a number to the next is obvious.]
- (ii) What number follows 93B?
- (iii) What number follows 9ABB?
- (c) Write down the place values of the digits in the base-12 number 2A09.
- (d) Use a method that relies on place values of digits to show how you would make the following base conversions:
- (i) 143 base 5 to base 10
- (ii) 143 base 10 to base 5
- (iii) 123.21 base 4 to base 10

QUESTION 3 (2 + 2 + 8 + 8 = 20)

- (a) In our Python programming we used structures called 'lists'. An example of a list might be $[5, 3, -2, 7.6, 3]$. What are the two differences between these 'lists' and our mathematical 'sets'?
- (b) Most of the sets we looked at were sets of numbers. Give a set of at least 4 members in which the **members** are not numbers, but where the members are somehow related (saying how they are related).
- (c) A useful set (sometimes called a 'power set') is a set consisting of a set and all of its subsets.
- (i) If a power set X contains $\{3,5,A\}$ and all its subsets – complete the following

$$X = \{ \{3,5,A\}, \dots \}$$
- (ii) The largest set in another power set Y has 5 members. What is the cardinality of Y ?
- (iii) Show how we could use 'Pascal's triangle' to find the number of subsets of size 3 in Y .
- (d) A bag consists of 40 marbles. Some are single coloured – either red (R), blue (B) or green (G). Others have multiple colours. Some are not coloured at all (they are clear). On counting a boy wrote down the number of marbles with various colourings – but then lost part of his list. See if you can complete it for him, using a venn diagram.

5 marbles were clear
 4 marbles had all three colourings R,B and G
 8 marbles had R and G colourings, but no B
 5 marbles had R and B colourings, but no G
 20 marbles had B amongst their colours
 18 marbles had G amongst their colours
 19 marbles had R amongst their colours

How many marbles were pure blue (B)?

QUESTION 4 (3 + 4 + 6 + 7 = 20 marks)

- (a) What would be printed (with reasons) by this Python program:

```
import math
v = math.sqrt(16)
print( v )
```

- (b) The when the following Python program is run the output is 21 . Use your knowledge of algebraic operations to explain carefully how this number is obtained (where '**' indicates 'raise to the power').

```
x = 4
x = 5-(x-2)**3+6*x
print( x )
```

- (c) Using the knowledge that '%' in Python means 'remainder after integer division', what (with reasons) would be printed by this program:

```
x = 23
result = 'odd'
if 4*x % 2 == 0 :
    result = 'even'
print( 4*x, 'is', result)
```

- (d) Consider this Python program:

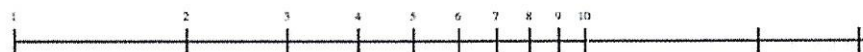
```
sum = 0
for i in range(1,5) :
    sum = sum + i*i
print( sum )
```

- (i) Which arithmetic sequence does 'range(1,5)' create?
- (ii) What would be printed (with reasons) if this program was run?

QUESTION 5 (4 + 4 + 4 + 4 + 4 = 20 marks)

- (a) Solve the following equations for x , checking that your answers are correct
- (i) $3x + 5 = 5(x - 3)$
- (ii) $x^3 - 7x^2 + 6x = 0$
- (b) Solve these inequalities
- (i) $3x - 4 \geq 17$
- (ii) $x^4 < 16$
- (c) Your solutions to (b) will split the number line into two or more parts. Select a number from each part and check that it agrees with your solution.
- (d) Solve this equation for x : $3 + 6^x = 8.3$
- (e) One use of logarithms is to change scaling.

- (i) The diagram below shows a (common) logarithmic number line.



What two numbers would be placed on the right two graduations on the above line?

- (ii) Show where the number 12 is placed on this line.
- (iii) A good example of the use of logarithmic scaling is in chemistry and pH values (the formula is $\text{pH} = -\log(\text{H}^+)$, and a typical pH value might be 5.8). Using pH values as an illustration, what are the two reasons why logarithms are used to rescale?

QUESTION 6 (3 + 3 + 4 + 5 + 5 = 20)

(a) Consider the identity: $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

(i) What do you understand by an 'identity'?

(ii) Formulate a direct proof of this identity.

(b) The following 'proof' contains an error, what is it?

$$(x - 3)^2 = x^2 - 3^2$$

$$\text{Let } x = 3$$

$$\text{LHS} = (3 - 3)^2 = 0$$

$$\text{RHS} = 3^2 - 3^2 = 0$$

LHS = RHS, therefore the first line is true.

(c) (i) How could you show that line 1 is false? Do it.

(ii) What, of course, should the correct line 1 in (c) be?

(d) (i) What is proof by exhaustion?

(ii) Use proof by exhaustion to show that $\overline{A \text{ and } B} = \overline{A} \text{ or } \overline{B}$

where A,B are either True or False, the overbar indicates negation,

A and B is true only if both A and B are true,

A or B is true if one or both A and B are true.

(e) The following is a 'proof by induction' – used to prove $1+2+3+ \dots +n = n(n+1)/2$

$$\text{Step 1: } 1 = 1 \times 2 / 2$$

$$\text{Step 2: } \text{if } 1 + 2 + \dots + j = j(j+1)/2$$

$$\text{then } 1 + 2 + \dots + j + (j+1) = j(j+1)/2 + (j+1)$$

$$= (j+1)(j/2 + 1) \quad *$$

$$= (j+1)(j+2)/2 \quad **$$

hence proved.

(i) Explain how the line labelled ** is obtained from line *

(ii) Using this example, what is induction? (ie, how does it work?)

----- End of Exam -----