THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
FIRST SEMESTER EXAMINATIONS 2023

BACHELOR OF SCIENCE IN APPLIED MATHEMATICS – FIRST YEAR

AM111 - FOUNDATION MATHEMATICS

TIME ALLOWED – 3 HOURS

INFORMATION FOR CANDIDATES:

1. Write your name, student number, and program of study clearly on the front page of your answer booklet. Do it **now**.
2. You have 10 minutes to read this examination paper. During this time you must **NOT** write **inside** your answer booklet. You can make notes on the examination paper.
3. A scientific calculator is permitted, though **you do not have to use one**.
   Other electronic devices are not permitted. Notes and headphones are not permitted.
4. At the conclusion of the examination you must **immediately** put your pens down. You are **NOT** permitted to write inside your answer booklet after the "end of examination" announcement.
5. You can answer the questions in any order. Start each question on a new page. After you have finished the exam, indicate the order in which you answered questions in the left column of the marks box on the cover of the answer booklet.
6. There are 6 questions. **You should attempt all questions.**

MARKING SCHEME:

Questions 1, 2 and 4 are worth 20 marks. Questions 2, 5 and 6 are worth 10 marks.
Total marks = 90.

QUESTION 1 (8 + 6 + 6 = 20 marks)

(a) (i) An example of a 4 element set might be $A = \{5, 11, 4, \text{blue}\}$
Of the various properties of sets, two are of critical importance.
What are they, and how do they affect the above set?

(ii) Two other sets are $B = \{11, \text{red}\}$ and $C = \{3, 4, \text{red}\}$
The universal set is the union of the three sets A, B and C.
In set notation, write down the elements of

(α) The universal set
(β) $A \cap B \cap C$
(χ) $A \cap (B \cup C)$
(δ) $A \cup (B \cap \overline{C})$ where $\overline{C}$ indicates the complement of set C

(b) A "power set" is a set that contains of all the subsets a given set.

(i) What is the power set that can be constructed from the set $\{1, 2, 3\}$

(ii) What is the formula that allows us to determine the number of elements that are in the power set generated from $\{1, 2, 3, 4, \ldots 9\}$

(iii) Use Pascal's triangle to show how we can deduce the number of subsets of size 3 are in the power set generated from $\{a, b, x, y, z\}$

(c) (i) Use a Venn diagram to illustrate the concept of the intersection of three sets.

(ii) Use an appropriate Venn diagram to answer this counting question:

64 students were asked which of the meals 'chicken with rice', 'chicken with noodles' or 'chicken with kaukau' they liked to eat.

3 said none.
3 said chicken with rice only
11 said chicken with rice and chicken with noodles (but no kaukau)
9 said chicken with rice and chicken with kaukau (but no noodles)
Overall 43 said they would eat chicken with rice,
45 would eat chicken with kaukau, and
45 would eat chicken with noodles.

How many students said they would eat only chicken with noodles?

QUESTION 2 (6 + 2 + 4 + 6 = 20 marks)

(a) Write down an example of each of a rational, irrational, integer, whole and natural number indicating the property that makes them this type of number.

(b) One of the theoretical rules of numbers is the "distributive rule of multiplication over addition"

(i) Give an example of this rule using three real numbers.

(ii) What is an important use of this rule in algebra?

(c) If we are checking that 593 is prime, we can proceed by dividing each smaller prime into it.

(i) Using this test, how do we deduce that 593 is or is not prime?
(Say how, but do not do it!)

(ii) In fact we only have to divide the primes up to 23. Explain why.

(d) We have a convention that distinguishes between "measurements" and "exact numbers".

(i) Explain the convention.

(ii) (α) What is the largest and smallest measurement that rounds to 7.348 (4sf)?
(β) Round 7.348 to 2sf.

(iii) To calculate an area, we need to multiply 7.348 by 11.3.
Using a calculator a student calculated this product to be 83.0324.
(α) What is the quick way to verify that this product could be correct?
(β) To what accuracy should the area be quoted, with a reason.

QUESTION 3 (4 + 4 + 2 = 10)

(a) A number in base 8 is 5041_8.

(i) What is the place value of each digit in this number.

(ii) If someone were to count out 5041_8 objects in base 8, then recount them in base 10, how many objects would there be (counting in base 10)?

(b) We used two methods to convert whole numbers from base 10 to a non-base 10 system. The first relied on understanding the meaning of base numbers, the second by applying a clever algorithm. Use both methods to convert 143_{10} to base 4.

(c) Convert the "decimal" hexadecimal number $B.40_{16}$ to decimal.
[The hex digits used are 0,1,2,…,9,A,B,C,D,E]

QUESTION 4 (6 + 6 + 4 + 4 = 20)

(a) Solve these equations for x:

(i) $3x + 12 = x^2 + 7x$

(ii) $x(2x + 1)^7 = x^8$

(iii) $4^x = 20$

(b) (i) How would you show that x=3 is a factor of $x^3 + x^2 - 17x + 15$?
Is x=3 a factor of this polynomial expression?

(ii) Simplify $(x+3)(2x+6)^3$

(iii) Simplify $\log(x) + \log(100x)$ where log is the common logarithm

(c) (i) Complete this definition: $\ln(a) = b$ if .... [where ln = natural logarithm]

(ii) Explain using (i) why taking the natural logarithm of a negative number is impossible.

(iii) Common logarithms are often used in measurement, and applied to very large measurements, or to positive measurements that are very close to 0.
Why? [Explain with an example]

(d) Solve these inequalities for x:

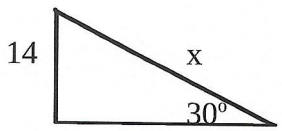
(i) $3 - 7x \leq x + 4$

(ii) $5/(2x - 1) > 3$

QUESTION 5 (3 + 5 + 2 = 10 marks)

(a) If α is an angle between 0° and 90°, draw two diagrams that illustrate how $\cos\alpha$ is defined using (i) a right angled triangle, and (ii) the unit circle.

(b) (i) Solve this right angled triangle for x:



(ii) Use your answer in (i), and some more calculations (at least one that involves Pythagoras' theorem), to find the perimeter of the triangle on the right which has height 14

(c) Use the addition formula $\cos\alpha+\beta = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
to derive the "double angle" formula $\cos 2\mu = 2\cos^2\mu - 1$

QUESTION 6 (2 + 2 + 2 + 4 = 10 marks)

(a) Asked to prove that $(a - b)^2 + 4ab = (a + b)^2$ a student proceeded like this:

Let a=5 and b=2.

LHS = $(5 - 2)^2 + 4 \times 5 \times 2$
= $3^2 + 40$
= 49

RHS = $(5 + 2)^2$
= 7^2
= 49

Since LHS = RHS the identity is true.

What is wrong with this proof?

(b) A second student gave this proof the same question:

	$(a - b)^2 + 4ab = (a + b)^2$		(1)
So	$(a - b)^2 + 4ab - (a + b)^2 = (a + b)^2 - (a + b)^2$		(subtract $(a+b)^2$ from both sides)
so	$(a - b)^2 + 4ab - ((a - b)^2 + 4ab) = 0$		(using (1))
so	$(a - b)^2 + 4ab - (a - b)^2 - 4ab = 0$		(remove brackets)
so	$0 = 0$		(simplifying LHS)

Since 0 = 0 is obviously true original the identity must be true.

What is wrong with this second proof?

(c) Give a correct direct proof for the identity $(a - b)^2 + 4ab = (a + b)^2$

Hint: Start with LHS = $(a - b)^2 + 4ab$

(d) Use mathematical induction to show that

$$10 + 20 + 30 + \ldots + 10n = 5n(n + 1)$$

-------- END OF EXAMINATION ---------