THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS - 2022

FIRST YEAR APPLIED MATHEMATICS

AM112 – CALCULUS AND ALGEBRA

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. **Answer any five (5) questions out of six (6) questions**.
4. All answers must be written in examination booklets only. No other written material will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes and textbooks are not allowed in the examination room. All mobile phones and electronic/recording devices must be switched off during the examination.
7. Scientific calculators are allowed in the examination room.
8. A formula sheet is attached.

MARKING SCHEME

Marks are indicated at the beginning of each question. All questions carry equal marks.

Question 1 FUNCTIONS AND LIMITS (10 marks)

a) Solve without using a calculator: find x if $\log_{10}(1+x) = 3$. (2 marks)

b) What is the amplitude of $f(x) = \frac{1}{2}\cos x$? (2 marks)

c) If $\sinh x_0 = 2$, what is $\cosh x_0$? (2 marks)

d) Find $\lim_{x\to 4} \frac{x^2-1}{x-4}$ (2 marks)

e) First rationalize the numerator, then find $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$ (2 marks)

Question 2 DERIVATIVES (10 marks)

a) Given that $x = 1 + \sin\theta$ and $y = \sin\theta - \frac{1}{2}\cos 2\theta$. Show that $\frac{d^2y}{dx^2} = 2$ (3 marks)

b) If $2x^2 + y^2 - 6y - 9x = 0$ determine the gradient of the normal to the curve at point $(1,7)$. (3 marks)

c) In the following function, find the coordinates of the relative extrema using the 1st and 2nd Derivative Test. $f(x) = 2x^3 - 9x^2 + 12x$ (4 marks)

Question 3 INTEGRATION (10 marks)

a) Solve the definite integral, $\int_{\pi/6}^{\pi/4} 5 - \sec x \tan x \, dx$ (3 Marks)

b) Solve $\int \sqrt{1+x^2}\, dx$ using trigonometric substitution. Draw the triangle to indicate the relevant working out. DO NOT USE ANY OTHER METHOD. Helpful table is given. The useful identity is $\sec^2\theta = 1 + \tan^2\theta$. (3 marks)

c) Find the volume V of the solid that is obtained when the region under the curve $y = \sqrt{9-x^2}$, over the interval $[-3,3]$ is revolved about the x-axis. Use the method of disks, hence use either $V = \int_a^b \pi[f(x)]^2 dx$ or $V = \int_c^d \pi[g(y)]^2 dy$. Sketch the volume, on the x-y axis (plane). (4 Marks)

Question 4 COMPLEX NUMBERS (10 marks)

a) Find and **plot all roots** of the following complex number: $w = \sqrt[3]{216}$. (7 marks)

b) Express each root in its rectangular, polar and exponential form. (3 marks)

Question 5 MATRICES (10 marks)

a) Given $\mathbf{A} = \begin{bmatrix} 0.5 & 0 & -0.5 \\ -0.1 & 0.2 & 0.3 \\ 0.5 & 0 & -1.5 \end{bmatrix}$,

(i) calculate the inverse from $\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}[A_{jk}]^T$. Where A_{jk} is the cofactor of a_{jk} in $\det \mathbf{A}$. (4 marks)

(ii) Check by using $\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$, (1 marks)

(b) Find the eigenvalues and eigenvectors of the following matrix, (Show all steps)

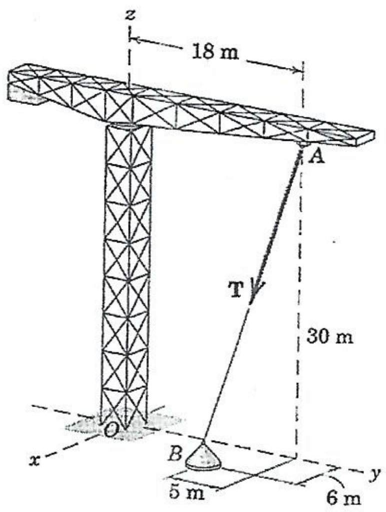
$$\begin{bmatrix} -6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$$

(5 Marks)

Question 6 VECTORS (10 marks)

In picking up a load from position B, a cable tension $\mathbf{T}$ of magnitude 24kN is developed. Calculate the moment which $\mathbf{T}$ produces about the base, O, of the construction crane.

You can solve using basic scalar algebra or basic vector algebra.



DATA SHEET for AM112 EXAMS 2022 SEMESTER 1

Trigonometrical identities

(a) $\sin^2\theta + \cos^2\theta = 1;\ \sec^2\theta = 1 + \tan^2\theta;\ \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(b) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(c) Let $A = B = \theta$ $\therefore$ $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Hyperbolic identities

$\cosh x + \sinh x = e^x$	$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$
$\cosh x - \sinh x = e^{-x}$	$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$
$\cosh^2 x - \sinh^2 x = 1$	$\sinh(x-y) = \sinh x\cosh y - \cosh x\sinh y$
$1 - \tanh^2 x = \operatorname{sech}^2 x$	$\cosh(x-y) = \cosh x\cosh y - \sinh x\sinh y$
$\coth^2 x - 1 = \operatorname{csch}^2 x$	$\sinh 2x = 2\sinh x\cosh x$
$\cosh(-x) = \cosh x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\sinh(-x) = -\sinh x$	$\cosh 2x = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$

Derivatives and Integrals

1 $\dfrac{d}{dx}(x^n) = nx^{n-1}$ $\therefore$ $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $\left\{\begin{matrix}\text{provided}\\ n \neq -1\end{matrix}\right\}$

2 $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$ $\therefore$ $\displaystyle\int \frac{1}{x}\,dx = \ln x + C$

3 $\dfrac{d}{dx}(e^x) = e^x$ $\therefore$ $\displaystyle\int e^x\,dx = e^x + C$

4 $\dfrac{d}{dx}(e^{kx}) = ke^{kx}$ $\therefore$ $\displaystyle\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$

5 $\dfrac{d}{dx}(a^x) = a^x\ln a$ $\therefore$ $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$

6 $\dfrac{d}{dx}(\cos x) = -\sin x$ $\therefore$ $\displaystyle\int \sin x\,dx = -\cos x + C$

7 $\dfrac{d}{dx}(\sin x) = \cos x$ $\therefore$ $\displaystyle\int \cos x\,dx = \sin x + C$

8 $\dfrac{d}{dx}(\tan x) = \sec^2 x$ $\therefore$ $\displaystyle\int \sec^2 x\,dx = \tan x + C$

More derivatives

$\dfrac{d}{dx}[\tan x] = \sec^2 x$ $\qquad$ $\dfrac{d}{dx}[\sec x] = \sec x\tan x$

$\dfrac{d}{dx}[\cot x] = -\csc^2 x$ $\qquad$ $\dfrac{d}{dx}[\csc x] = -\csc x\cot x$

Specific integrals

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

TRIGONOMETRIC SUBSTITUTIONS

EXPRESSION IN THE INTEGRAND	SUBSTITUTION	RESTRICTION ON θ	SIMPLIFICATION
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$-\pi/2 \le \theta \le \pi/2$	$a^2 - x^2 = a^2 - a^2\sin^2\theta = a^2\cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$-\pi/2 < \theta < \pi/2$	$a^2 + x^2 = a^2 + a^2\tan^2\theta = a^2\sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\begin{cases} 0 \le \theta < \pi/2 & (\text{if } x \ge a) \\ \pi/2 < \theta \le \pi & (\text{if } x \le -a) \end{cases}$	$x^2 - a^2 = a^2\sec^2\theta - a^2 = a^2\tan^2\theta$

Reduction formula

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$$

$$\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$

INTEGRATING PRODUCTS OF TANGENTS AND SECANTS

$\int \tan^m x \sec^n x \, dx$	PROCEDURE	RELEVANT IDENTITIES
n even	• Split off a factor of $\sec^2 x$. • Apply the relevant identity. • Make the substitution $u = \tan x$.	$\sec^2 x = \tan^2 x + 1$
m odd	• Split off a factor of $\sec x \tan x$. • Apply the relevant identity. • Make the substitution $u = \sec x$.	$\tan^2 x = \sec^2 x - 1$
$\begin{cases} m \text{ even} \\ n \text{ odd} \end{cases}$	• Use the relevant identities to reduce the integrand to powers of $\sec x$ alone. • Then use the reduction formula for powers of $\sec x$.	$\tan^2 x = \sec^2 x - 1$

- **Roots of a complex number:** $\sqrt[n]{z} = \left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$ where; $k = 0, 1, 2, \cdots$

- **Eigenvalues and eigenvectors:**
  $\mathbf{Ax} = \lambda\mathbf{x}$ & $(\mathbf{A} - \lambda\mathbf{I}) = \mathbf{0}$

- **In vector algebra:** $\vec{T} = T\vec{n} = T\frac{\overrightarrow{AB}}{\overline{AB}}$