THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS – 2022

FIRST YEAR BACHELOR OF SCIENCE IN APPLIED MATHEMATICS

AM 125 – LINEAR ALGEBRA I

TIME ALLOWED: 3 HOURS

INSTRUCTIONS FOR CANDIDATES:

1. You have 10 minutes to read through this paper. You must **NOT** begin writing during this time.
2. There are five (5) questions. Answer **ALL** questions.
3. Write all answers in the answer booklet(s) provided.
4. All workings should be shown clearly in the answer booklet(s).
5. Start each question on a new page and clearly write its question number at the top of the page.
6. Calculators are allowed in the examination room.
7. Mobile phones **must** be switched off during the examination period.
8. Make sure that your **name**, **surname** and **ID number** are clearly written on the front of the examination answer booklet(s).
9. Required formulas are provide at the end of the question paper.

MARKING SCHEME

Questions carry marks as indicated. Total marks: **75**

Question 1: $[3 + (2 + 3) = 8$ **marks**$]$

(a) How do we determine whether a set is a vector space or not.

(b) What is a subspace? List the three properties that defines a subspace.

Question 2: $[5 + 5 + 3 = 15$ **marks**$]$:

(a) Define dot product in 2-space.
(b) Prove the commutative property of dot product of vectors in 3-space.
(c) Explain why **a•b•c** is meaning less.

Question 3: $[(3 + 1) + (1 + 3 + 3 + 8) = 19\ \boldsymbol{marks}]$

(a) What is a triangular matrix? Give an example of a 3x3 triangular matrix.

(b) Given the system $\begin{cases} x + y + z = 6 \\ x + 2y + 3z = 10 \\ x + 2y - z = 12 \end{cases}$

(i) Form the augmented coefficient matrix.
(ii) Use elementary row operation to reduce it to triangular matrix form.
(iii) Determine the rank of the matrix.
(iv) Using the result of part(ii) above, solve the system.
(v) Prove the solution obtain in part (iv) above using Cramer's Rule.

Question 4: $[1 + 3 + 3 + 8 = 15\ \boldsymbol{marks}]$

Given $A = \begin{bmatrix} -1 & -3 & 1 \\ 3 & 6 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(i) What type of matrix is matrix A.
(ii) Explain the relationship between <u>determinant</u> and <u>inverse</u> of a sqaure matrix?
(iii) Is it true that all square matrices are invertible? Explain your answer.
(iv) Find the inverse of matrix A using the formula $\boldsymbol{A^{-1} = \frac{1}{|A|}\mathrm{adj}(A)}$.

Question 5: $[4 + 3 + (2 + 3 + 3) + 8 = 23\ marks]$

(a) Find $\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix}$ and simplify.

(b) Given $\mathbf{a} = \begin{pmatrix} 3 \\ 3-2t \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} t^2+t \\ -2 \end{pmatrix}$, determine the value of t if **a** and **b** are perpendicular.

(c) For $\mathbf{p} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ find:

(i) $\mathbf{p}\cdot\mathbf{q}$ (ii) $\| \mathbf{p} + 2\mathbf{q} - \mathbf{r} \|$ (iii) the angle between **p** and **r.**

(d) Determine the eigenvalues and eigenvectors for the equations $Ax = \lambda x$ where $A=\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.

END OF EXAMINATION

Formula Sheet

Name of Rule	Formula
Cramer's Rule	$x_1 = \frac{Dx_1}{D}, x_2 = \frac{Dx_2}{D}, \dots, x_n = \frac{Dx_n}{D}$
Inverse Matrix	$A^{-1} = \frac{1}{\lvert A \rvert} adj(A)$
Norm of vector	$\lVert \mathbf{u} \rVert = \sqrt{u_1{}^2 + u_1{}^2 + \dots + u_n{}^2}$
Dot product	$\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2 + \dots + u_nv_n$
Characteristic equation	$\lvert A - \lambda I \rvert = 0$