THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS – 2022

FIRST YEAR BACHELOR IN APPLIED MATHEMATICS

AM126 – ANALYTICAL GEOMETRY

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIVE (5) questions. You need to **answer ALL** the questions.
4. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.
8. The last two pages contains formula sheet for student information.

MARKING SCHEME

Marks are indicated at the beginning of each question. The total is **100 marks with 50% weightage.**

QUESTION 1 [5 + 5 + (3 +3 +2 + 2) = 20 marks]

(a) Find an equation of the line through the point $P(2,3)$ which forms an isosceles triangle with the coordinate axes in the first quadrant.

(b) The xy-coordinate axes are rotated about the origin through an angle of 45^0. If xy-coordinates of a point are $(5,1)$, find its XY-coordinates, where OX and OY are the axes obtained after rotation.

(c) During braking, the velocity of a vehicle satisfies the relation $v(t) = v_0 + \alpha t$;
where $v_0 \equiv$ initial velocity (m/s), $\alpha \equiv acceleration\,(m/s^2)$.
Determine the initial velocity v_0 and the acceleration α if the velocity is known at the following two points. Also find the total stopping time and hence sketch the graph.

$t(s)$	$v(t)\ (m/s)$
0.75	35
1.25	2.4

QUESTION 2 [10 + 10 = 20 marks]

(a) Find a joint equation of the straight lines through the origin and perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$.

(b) Find the measure of the angle between the lines represented by $x^2 - xy - 6y^2 = 0$.

QUESTION 3 [5 + (6 +4) + 5 = 20 marks]

(a) Find the foot N of the perpendicular drawn from $P(-2,7,-1)$ to the plane $2x - y + z = 0$.

(b) Determine the length and equation of the shortest distance line between the lines:
$l_1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $l_2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

(c) Find the equation of the sphere for which the circle
$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y + 4z = 8$ is a great circle.

QUESTION 4 [4 + 6 + (2 +1 +2 + 3) +2 = 20 marks]

(a) Examine the nature of the conic $2x^2 + 3y^2 = 0$.

(b) Determine the centre, radius, x-intercept, y-intercept and parametric equations of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$.

(c) For a given ellipse $\frac{x^2}{4}+\frac{y^2}{49}=1$, determine the following:

(i) Equations of major and minor axis.
(ii) Equations of directrices
(iii) Length of major and minor axis.
(iv) Vertices, foci, length of latus rectum.

(d) For a given hyperbola $\frac{x^2}{36}-\frac{y^2}{64}=1$, find its equations of transverse and conjugate axis.

QUESTION 5 [(2 +2 + 3 + 3) + (8 + 2) = 20 marks]

(a) For a given point $P(-2,6,3)$ and vector $A = ya_x + (x+z)a_y$; express 'P' and 'A' in cylindrical and spherical coordinates. Evaluate 'A' at 'P' in the Cartesian and cylindrical systems.

(b) Express the vector $B = \frac{10}{r}b_r + r\cos\theta b_\theta + b_\phi$ in Cartesian coordinate system and hence evaluate $B(-3,4,0)$.

END OF EXAM

FORMULAE SHEET

Coordinates of Internal/external division:	$\left(\frac{k_1x_2 \pm k_2x_1}{k_1 \pm k_2}, \frac{k_1y_2 \pm k_2y_1}{k_1 \pm k_2}\right)$
Translation:	$X = x - h, Y = y - k$
Rotation:	$X = x\cos\theta + y\sin\theta, Y = y\cos\theta - x\sin\theta$
Symmetric form of straight line:	$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r$
Normal form of straight line:	$x\cos\alpha + y\sin\alpha = p$
Angle between two lines:	$\theta = \tan^{-1}\left\lvert\frac{m_2 - m_1}{1 + m_1m_2}\right\rvert$
Area of a triangle:	$\frac{1}{2}\lvert x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\rvert$
For a pair of straight lines:	$ax^2 + 2hxy + by^2 \equiv \left(by + hx + x\sqrt{h^2 - ab}\right)\left(by + hx - x\sqrt{h^2 - ab}\right)$
Angle between pair of straight lines:	$\theta = \tan^{-1}\left\lvert\frac{2\sqrt{h^2 - ab}}{a + b}\right\rvert$
Angle between two 3D lines:	$\theta = \cos^{-1}\lvert l_1l_2 + m_1m_2 + n_1n_2\rvert$; $l_1, m_1, n_1; l_2, m_2, n_2$ are the D.C's
Angle between two 3D lines:	$\theta = \cos^{-1}\left\lvert\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right\rvert$; $a_1, b_1, c_1; a_2, b_2, c_2$ are the D.R's
Angle between a 3D line and a plane:	$\theta = \sin^{-1}\left\lvert\frac{al + bm + cn}{\sqrt{a^2 + b^2 + c^2}\sqrt{l^2 + m^2 + n^2}}\right\rvert$; l, m, n are the D.R's of the line.
Internal/external point of contact between two spheres:	$\left(\frac{-r_1g_2 \pm r_2g_1}{r_1 \mp r_2}, \frac{-r_1f_2 \pm r_2f_1}{r_1 \mp r_2}, \frac{-r_1c_2 \pm r_2c_1}{r_1 \mp r_2}\right)$
Tangent line to a given sphere:	$xx_1 + yy_1 + zz_1 + g(x + x_1) + f(y + y_1) + c(z + z_1) + d = 0$

Forms of Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Axis:	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix:	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex:	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Focus:	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Length of l.r.	4a	4a	4a	4a
Focal length:	$\lvert x + a\rvert$	$\lvert x - a\rvert$	$\lvert y + a\rvert$	$\lvert y - a\rvert$

Forms of Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; a^2 > b^2$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1; a^2 > b^2$
Vertex:	$(\pm a,0)$	$(0,\pm a)$
Focus:	$(\pm ae,0)$	$(0,\pm ae)$
Directrix:	$x = \pm a/e$	$y = \pm a/e$

Major Axis/Minor Axis	$y=0$; $x=0$	$x=0$; $y=0$
Length of major axis:	2a	2a
Length of minor axis:	2b	2b
Length of l.r.	$2b^2/a$	$2b^2/a$
Ecentricity:	$\sqrt{a^2-b^2}/a$	$\sqrt{a^2-b^2}/a$
Forms of hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Foci:	$(\pm c,0)$; $c^2=a^2+b^2$	$(0,\pm c)$; $c^2=a^2+b^2$
Ecentricity:	$e=\frac{c}{a}>1$	$e=\frac{c}{a}>1$
Vertices:	$(\pm a,0)$	$(0,\pm a)$
Slope of asymptotes:	$\pm\frac{b}{a}$	$\pm\frac{a}{b}$
Directrix:	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$
Length of l.r.:	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Transverse axis:	$y=0$	$x=0$
Conjugate axis:	$x=0$	$y=0$

Cartesian to cylindrical coordinates:	$\rho=\sqrt{x^2+y^2}, \phi=\tan^{-1}\left(\frac{y}{x}\right), z=z$
Cylindrical to Cartesian coordinates:	$x=\rho\cos\phi, y=\rho\sin\phi, z=z$
Cartesian to spherical coordinates:	$r=\sqrt{x^2+y^2+z^2}, \theta=\tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right), \phi=\tan^{-1}\left(\frac{y}{x}\right)$
Spherical to Cartesian coordinates:	$x=r\sin\theta\cos\phi, y=r\sin\theta\sin\phi, z=r\cos\theta$
Cartesian to cylindrical transformation:	$\begin{pmatrix}A_\rho\\A_\phi\\A_z\end{pmatrix}=\begin{pmatrix}\cos\phi & \sin\phi & 0\\-\sin\phi & \cos\phi & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}A_x\\A_y\\A_z\end{pmatrix}$
Cartesian to Spherical transformation:	$\begin{pmatrix}A_r\\A_\theta\\A_\phi\end{pmatrix}=\begin{pmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\\-\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta\\-\sin\phi & \cos\phi & 0\end{pmatrix}\begin{pmatrix}A_x\\A_y\\A_z\end{pmatrix}$
Spherical to Cartesian transformation:	$\begin{pmatrix}A_x\\A_y\\A_z\end{pmatrix}=\begin{pmatrix}\sin\theta\cos\phi & \sin\theta\cos\phi & -\sin\theta\\\sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi\\\cos\theta & -\sin\theta & 0\end{pmatrix}\begin{pmatrix}A_r\\A_\theta\\A_\phi\end{pmatrix}$