



PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SEMESTER ONE EXAMINATIONS – 2023

SECOND YEAR BACHELOR OF SCIENCE IN APPLIED MATHEMATICS (BSAM/2)

AM211 – CALCULUS AND ANALYSIS

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

- 1 You have 10 minutes to read this paper. You must not begin writing during this time.
- 2 Write your name and student number clearly on the front of the examination answer booklet.
- 3 There are 8 questions. You should attempt ALL questions.
- 4 All answers must be written in the examination answer booklet(s) provided. No other written material will be accepted.
- 5 Start the answer for each question on a new page. Do not use red ink or pencil.
- 6 Notes and textbooks are not allowed in the examination room.
- 7 Mobile phones and other recording devices are not allowed in the examination room.

MARKING SCHEME

Marks are as indicated at the beginning of each question. Total mark is 100.

QUESTION ONE [5 + 5 = 10 Marks]

Evaluate the following limits.

a. $\lim_{(x, y, z) \rightarrow (4, 1, -3)} \frac{x^2 y - 3z}{2x + 5y - z}$ b. $\lim_{(x, y, z) \rightarrow (4, -1, 3)} \sqrt{13 - x^2 - 2y^2 + z^2}$.

QUESTION TWO [8 + 8 = 16 Marks]

Given the function $f(x, y) = e^x \cos y + e^y \sin x$, find f_x and f_y .

QUESTION THREE [13 Marks]

Find the equation of the tangent plane to the surface defined by the function

$f(x, y) = 2x^2 - 3xy + 8y^2 + 2x - 4y + 4$ at point $(2, -1)$.

QUESTION FOUR [12 Marks]

Evaluate the double integral over the region R .

$$\iint_R (x \sin y - y \sin x) dA; \quad R = \{(x, y) : 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/3\}$$

QUESTION FIVE [12 Marks]

Evaluate the integral $\iint_R 3x dA$ over the region $R = \{(r, \theta) | 1 \leq r \leq 2, 0 \leq \theta \leq \pi\}$.

Hint: Use the conversion $x = r \cos \theta$ and $dA = r dr d\theta$

QUESTION SIX [15 Marks]

Evaluate the triple integral $\iiint_B z \sin x \cos y dV$ where $B = \{(x, y, z) | 0 \leq x \leq \pi, \frac{3\pi}{2} \leq y \leq 2\pi, 1 \leq z \leq 3\}$.

QUESTION SEVEN [12 Marks]

Find the gradient vector field of the function $f(x, y) = x \sin y + \cos y$

QUESTION EIGHT [10 Marks]

Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = 8x^2 y z \vec{i} + 5z \vec{j} - 4x y \vec{k}$ and C is the curve given by $\vec{r}(t) = t \vec{i} + t^2 \vec{j} + t^3 \vec{k}$, $0 \leq t \leq 1$.

END OF EXAM