THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS – 2023

SECOND YEAR BACHELOR OF SCIENCE IN APPLIED MATHEMATICS

AM 212 – LINEAR ALGEBRA II

TIME ALLOWED: 3 HOURS

INSTRUCTIONS FOR CANDIDATES:

1. You have 10 minutes to read through this paper. You must **NOT** begin writing during this time.
2. There are four (4) questions. Answer **ALL** questions.
3. Write all answers in the answer booklet(s) provided.
4. All workings should be shown clearly in the answer booklet(s).
5. Start each question on a new page and clearly write its question number at the top of the page.
6. Calculators are allowed in the examination room.
7. Mobile phones **must** be switched off during the examination period.
8. Make sure that your **name, surname** and **ID number** are clearly written on the front of the examination answer booklet(s).
9. Required formulas are provide at the end of the question paper.

MARKING SCHEME

Questions carry marks as indicated. Total marks: 80

Question 1: $[(5+3+3)+6+5=22\ \textbf{marks}]$

(a) Fully define the following terms or expressions:
   (i) Vector space.
   (ii) Dimension of a vector space.
   (iii) Span of vectors.

(b) What is a subspace? List the three properties that defines a subspace.
(c) In order for a set of vectors to be called a basis, it must satisfy two conditions. What are they? Describe any one of the two.

Question 2: $[6+4+(3+4)=17\ \textbf{marks}]$

(a) Consider these two statements in vector space:
"Orthogonal implies orthonormal" and "Orthonormal implies orthogonal".

Which of these statements is correct? Explain why one is correct and not the other.

(b) In order for a basis to be called an orthonormal basis, it must satisfy four conditions. List them.
(c) Given the basis set $\{(1,3)(-3,1)\}$ in $\mathbb{R}^2$;
   (i) show that the set is orthogonal basis but not an orthonormal basis.
   (ii) Convert it to an orthonormal basis.

Question 3: $[4+(4+5)+5+8=29\ \boldsymbol{marks}]$

(a) Given $\boldsymbol{p}=3-x+2x^2$ and $\boldsymbol{q}=4x+x^2$ are vectors in inner product space P_2. Calculate the distance between $\boldsymbol{p}$ and $\boldsymbol{q}$.
(b) Consider the inner product space $C[0,1]$ with the usual inner product
$\langle f,g\rangle=\int_a^b f(t)g(t)dt$.
Compute:
   (i) $\langle 1+t^2,4t\rangle$.
   (ii) $\|1+t^2\|$

(c) Given that $\boldsymbol{A}=4\boldsymbol{i}-\boldsymbol{j}-2\boldsymbol{k}$ and $\boldsymbol{B}=\langle 2,-2,4\rangle$, determine to two decimal places, the measure of the angle between the two vectors.
(d) Given $\boldsymbol{u}=\begin{bmatrix}-3\\-5\\2\end{bmatrix}, \boldsymbol{v}=\begin{bmatrix}4\\5\\-6\end{bmatrix}$, Show that $\|\boldsymbol{u}+\boldsymbol{v}\|^2=\|\boldsymbol{u}\|^2+\|\boldsymbol{v}\|^2-2\,\boldsymbol{u}\bullet\boldsymbol{v}$.

Question 4: $[6 + 6 = 12\ marks]$

Given $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$, find the basis, dimension and rank for

(i) Column space of A.

(ii) Null space of A.

END OF EXAMINATION

Formula Sheet

Name of Rule	Formula
Norm of vector	$\|\mathbf{u}\| = \sqrt{u_1{}^2 + u_1{}^2 + \cdots + u_n{}^2}$
Dot product	$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$
Commutative	$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
Convert to orthonormal	$u = \frac{v}{\|v\|}$
Angle between 2 vectors	$\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|v\|}$