



THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY  
DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE  
**FIRST SEMESTER EXAMINATIONS – 2023**  
**SECOND YEAR BACHELOR OF APPLIED MATHEMATICS**

**AM213 – DISCRETE MATHEMATICS**

**TIME ALLOWED: 3 HOURS**

**INFORMATION FOR CANDIDATES**

1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIFTEEN (15) questions. You are to **answer ALL** the questions.
4. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
5. Start the answer for each question on a **new page**. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.

**MARKING SCHEME**

Marks are indicated on the right side of each question. The total is **90 marks**.

**SECTION 1: MULTIPLE CHOICE QUESTIONS (2 marks each = 10 marks)**

**For questions 1 to 5: Select the answer that you believe is the most accurate or appropriate response to the question.**

1. Which of the following is NOT an example of a counting principle in mathematics?
  - a. The product rule
  - b. The addition rule
  - c. The pigeonhole principle
  - d. The Pythagorean theorem
  
2. How many ways can the letters of the word "MATHEMATICS" be arranged if all the letters are used?
  - a.  $11!$  (factorial)
  - b.  $11P4$  (permutation)
  - c.  $11C4$  (combination)
  - d.  $11^4$  (power)
  
3. Which of the following represents the linear difference equation for the sequence  $\{55, 50, 45, 40, \dots\}$ .
  - a.  $y_{n+1} = y_{n-1} - 5$ , where  $y_1 = 55$
  - b.  $y_n = 55 - 5$ , where  $y_1 = 55$
  - c.  $y_{n+1} = y_n - 5$ , where  $y_1 = 55$
  - d.  $y_n = y_{n+1} - 5$ , where  $y_1 = 55$
  
4. Which of the following is a requirement for a set with an operation to form a group?
  - a. Associativity
  - b. Commutativity
  - c. Existence of identity element
  - d. Closure under the operation

5. Which of the following statements accurately describes a mathematical ring homomorphism?
- A ring homomorphism preserves the multiplicative identity of the ring.
  - A ring homomorphism preserves the additive identity of the ring.
  - A ring homomorphism always results in a commutative ring.
  - A ring homomorphism is an isomorphism between two rings.

**SECTION 2: SHORT ANSWER QUESTIONS (32 marks)**

1. Carefully read and provide your answers for the following questions on the combination counting technique.
- Define a combination.  
*(3 marks)*
  - Explain how it differs from a permutation.  
*(4 marks)*
  - Provide an example of a combination.  
*(3 marks)*
2. Clearly outline the steps of the Euclidean algorithm for finding the greatest common divisor (GCD) of two numbers.  
*(8 marks)*
3. Explain the concept of proof by induction.  
*(6 marks)*
4. Carefully read and answer the following questions on mathematical subgroups.
- What is a subgroup?  
*(5 marks)*
  - Provide an example of a subgroup  
*(3 marks)*

**SECTION 3: PROBLEM SOLVING QUESTIONS (48 marks)**

1. A committee of twelve (12) is to be selected from ten (10) men and ten (10) women.  
In how many ways can the selection be carried out if;

a. there are no restrictions?

*(4 marks)*

b. there must be six men and six women?

*(5 marks)*

2. Show by direct proof that the product of two odd integers is odd.

*(6 marks)*

3. Express  $\gcd(2260, 816)$  in the form  $2260m + 816n$  where  $m, n \in \mathbb{Z}$ .

**(Show all your working out to get full marks).**

*(10 marks)*

4. Below is a relation  $R$  on a set  $A$ , where  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ :

$aRb$  if and only if  $a < b$ .

a. Draw the directed graph of the relation.

*(3 marks)*

b. Draw the binary matrix of the relation.

*(3 marks)*

5. Suppose there exists a relation  $R$  on  $A$ , where  $A = \{a, b, c, d, e\}$   
and  $R = \{(a, b), (b, c), (c, d), (d, e), (e, a)\}$ .

a. Is  $R$  reflexive? **Do justify your answer.**

*(3 marks)*

b. Is  $R$  symmetric? **Do justify your answer.**

*(3 marks)*

c. Is  $R$  transitive? **Do justify your answer.**

*(3 marks)*

6. The following is a Cayley table for a binary operation (\*) on the set {A, B, C, D, E, F}.

*	A	B	C	D	E	F
A	A	B	C	D	E	F
B	B	C	A	F	D	E
C	C	A	B	E	F	D
D	D	E	F	A	B	C
E	E	F	D	C	A	B
F	F	D	E	B	C	A

- a. Is the binary operation (\*) commutative? **Justify your answer.**

*(4 marks)*

- b. Is the binary operation (\*) associative? **Justify your answer.**

*(4 marks)*

THE END