THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING -2nd YEAR DEGREE

FIRST SEMESTER EXAMINATIONS - 2022

CE 211 – INTRODUCTION TO STRUCTURES

DATE: TUESDAY, 7TH JUNE 2022 – 08:20 A.M

VENUE: STRUCTURES LECTURE THEATRE (SLT)

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. You have 10 minutes to read the paper before the examination starts. You must **not** begin writing during this time.
2. **There are EIGHT (8) Questions in this paper. Answer any FOUR (4) questions which shall give a total of 80 points.**
3. Use only ink. Do not use pencils for writing except for drawings and sketches.
4. Only Calculator is allowed in the examination room. MOBILE PHONE is not allowed **(Switch your Mobile Phones OFF)**. Notes and textbooks are not allowed.
5. Start each question on a new page and show all your calculations in the answer book provided. No other material will be accepted.
6. **Write your NAME and Student NUMBER clearly on the front page. Do it now.**
7. **Marking Scheme:** All questions carry equal marks.

Question One [Member Forces] **(20 marks)**

a) Discuss the Method of Joints and the Method of Sections and state their advantages or disadvantages in determining internal member forces of a truss or frame. (4 Marks)

b) Using the method of sections, write the equations and plot the shear force and bending moment diagrams for the cantilever beam shown in **Figure Q1B**. Indicate the salient values. (8 Marks)

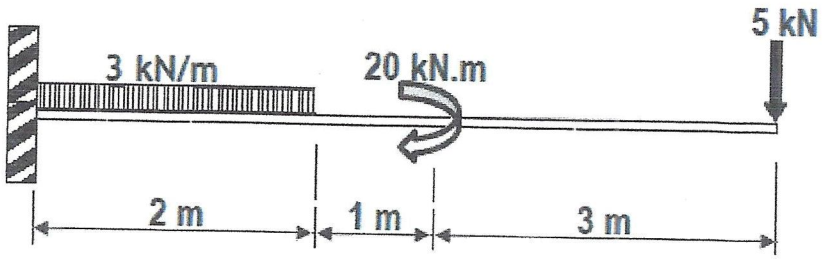


Figure Q1B

c) Determine the internal member forces of members **AE, BC** and **DE,** using either the Method of Joints or the Method of Sections, in the truss shown below in **Figure Q1C**. (8 Marks)

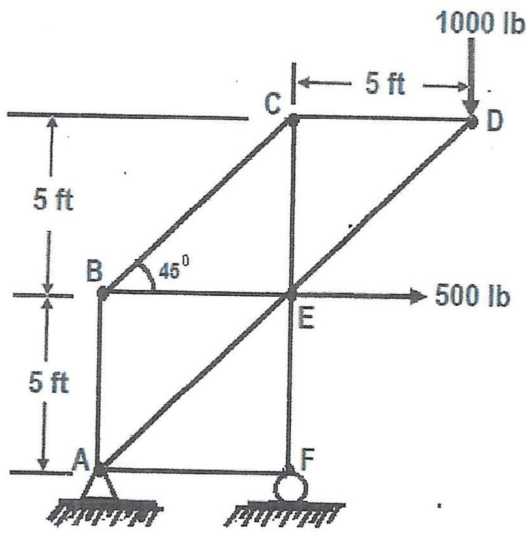


Figure Q2C

Question Two [Influence Lines] **(20 marks)**

Using the Mueller Model Principle, for the statically determinate beam shown in **Figure Q2**;

a) Sketch the Influence Lines for the reactions: **R1, R2,** and **R3.**
b) Sketch the Influence Lines for; (i) Shear force and (ii) bending moment at the point **B**, and
c) Determine the shear force and bending moment at point B as a uniformly distributed load of intensity **5k/m** and length **4 m** passes from left to right on the beam.

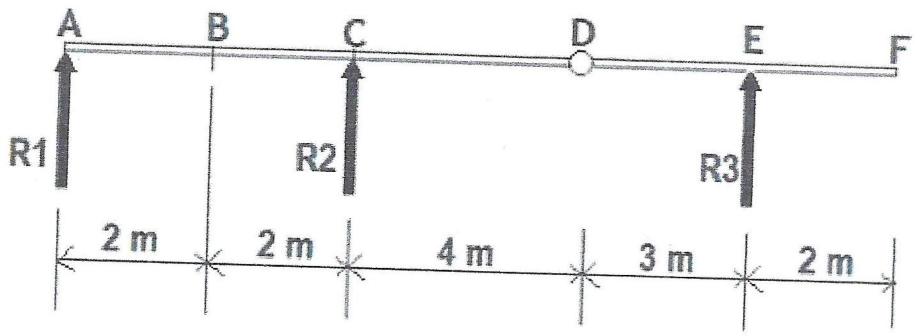


Figure Q2

Question Three [Three-pinned Arch] **(20 marks)**

a) What is the difference between a two-pinned arch and a three-pinned arch? (4 marks)
b) A **circular** arch ABCD **(Radius = 20 m)** is shown in **Figure Q3** together with all dimensions. Find the support reactions at the abutments/supports **A** and **B**, and the bending moments at **D** when the arch supports a uniformly, horizontally distributed load of intensity 12 kN/m on the entire span. Also draw the bending moment diagram for the arch. (16 marks)

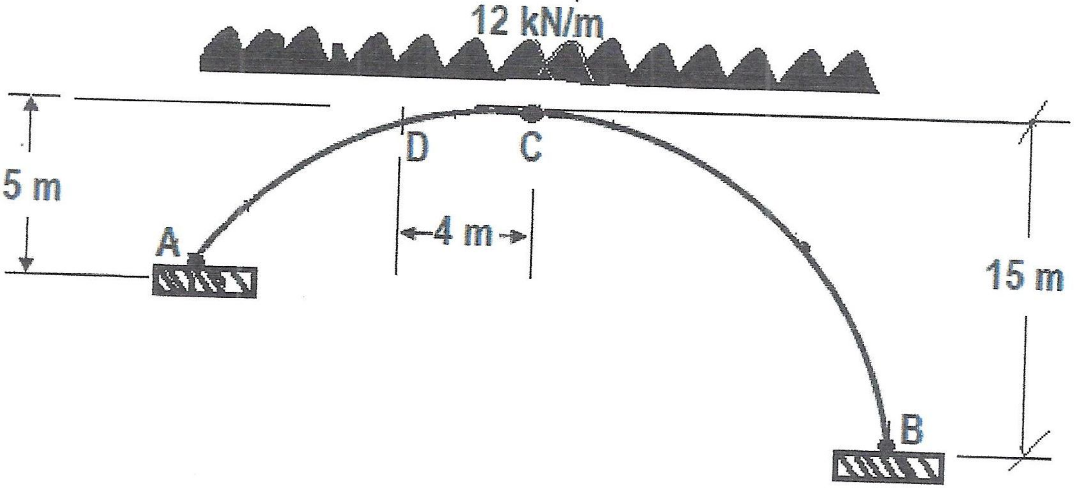


Figure Q3

Question Four [Singularity Functions] **(20 marks)**

(a) Briefly discuss what singularity functions are and state why it is necessary for solving beam loadings. (4 Marks)

(b) Using Singularity Functions, express the shear forces and bending moment in the beam **ABCDE** shown in **Figure Q4,** and draw the shear force and bending moment diagrams for the beam and indicate principal values. Loads and dimensions are indicated in the diagram. (16 Marks)

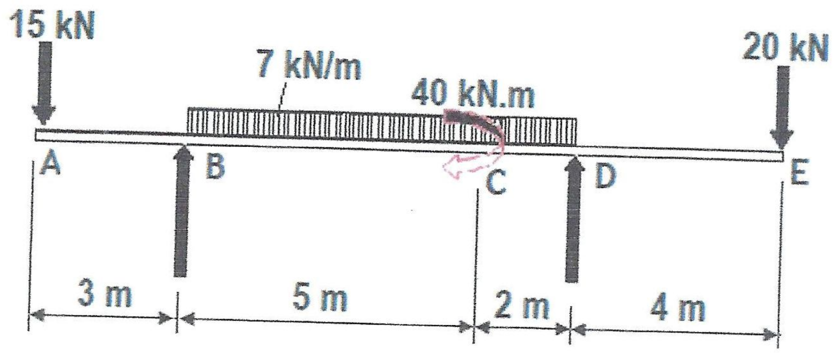


Figure Q4

Question Five [Torsion of Circular Shaft] **(20 marks)**

(a) A solid shaft **127 mm** diameter transmits **600 kW at 300 r.p.m.** It is also subjected to a bending moment of **9.1 kN.m** and an end thrust. If the maximum principal stress is limited to **77 N/mm²** find the end thrust. (8 Marks)

(b) A hollow steel shaft having outside and inside diameters of **45 mm** and **19 mm** respectively is subjected to a gradually increasing axial torque. The yield stress is reached at the surface when the torque is **1 kN.m**, the angle of twist per meter length then being **2.43°.** Find the magnitude of the yield stress. (12 Marks)

If the torque is increased to 1.08 kN.m, calculate;
(a) the depth to which yielding will have penetrated,
(b) the angle of twist per meter length. State any assumptions made and prove and special formula used.

Question Six [Shear/Bending Stress in Beams] **(20 marks)**

(a) An upside-down T-section beam symmetrical about a vertical axis, is made with a bottom flange 120 mm wide and 20 mm thick to which a vertical web plate, 10 mm wide and 200 mm deep is welded as in **Figure Q6**. At a certain section, where the total shearing force is **40 kN**:

(i) Calculate the shear stresses at the following points on the cross-section: Top of the section, the neutral axis, junction of the flange and web, and bottom of the section

(ii) Plot the shear stress distribution diagram. (10 Marks)

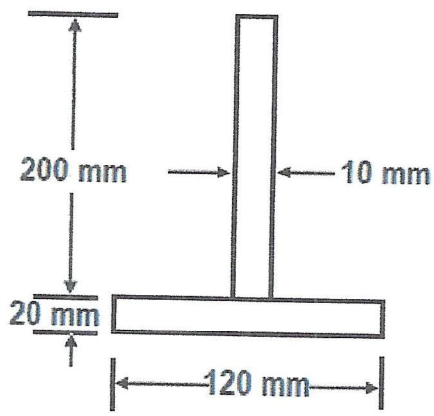


Figure Q6A

(b) A uniform T-section beam is **100 mm** wide and **150 mm** deep as shown in **Figure Q6B**. If the limiting bending stresses for the material of the beam are **80 N/mm²** in compression and **160 MN/m²** in tension, find the maximum uniformly distributed load (udl) **w**, that the simply supported beam can carry over a length of **3 m**. (10 Marks)

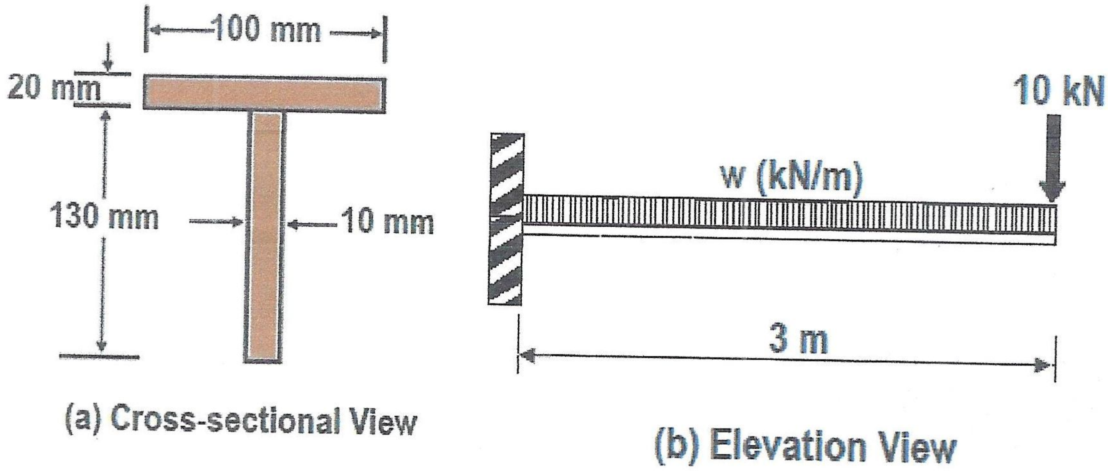


Figure Q6B

Question Seven [Composite Beam Sections] **(20 marks)**

(a) Briefly state one reason why a composite beam section may be necessary in design. (3 Marks)

(b) A timber beam 150 mm x 75 mm is reinforced by two 12 mm thick steel plates at top and bottom as shown in **Figure Q7B**. The composite beam is 3 m long, simply supported at each end, and carries a point Isteral load of 10 kN located at 1 m from the left hand support.
(i) Calculate the maximum bending stresses in the steel and timber.
(ii) Plot the bending stress distribution.
Take Modular ration, $E_S/E_T = 20$. (10 Marks)

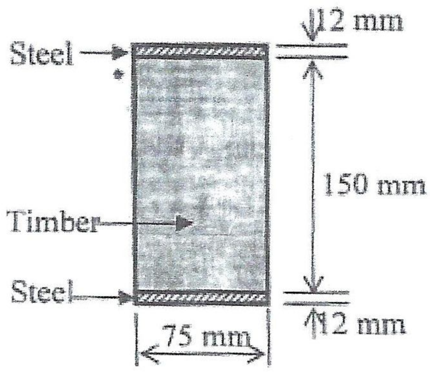


Figure Q7B

(c) A timber beam is reinforced by a Steel plate as shown in **Figure Q7C**. Transform the two materials into a timber beam section and calculate the maximum stresses in the two materials for a bending moment of 10 kN.m, and plot the stress (bending) distribution. Note: Modular Ration, $E_S/E_T = 10$. (7 Marks)

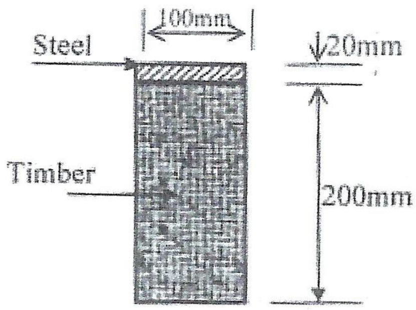


Figure Q7C

Question Eight [Stress-Strain Relationship] **(20 marks)**

The following figures were obtained in a standard tensile test on a specimen of low carbon steel:

Diameter of the specimen, 11.28 mm
Gauge length, 56 mm
Minimum diameter after fracture, 6.45 mm

Load (kN)	2.47	4.97	7.4	9.86	12.33	14.8	17.27	19.74	22.2	24.7
Extension (m x 10^{-6})	5.6	11.9	18.2	24.5	31.5	38.5	45.5	52.5	59.5	66.5
Load (kN)	27.13	29.6	32.1	33.3	31.2	32	31.5	32	32.2	34.5
Extension (m x 10^{-6})	73.5	81.2	89.6	112	224	448	672	840	1120	1680
Load (kN)	35.5	37	38.7	39.5	40	39.6	35.7	28		
Extension (m x 10^{-6})	1960	2520	3640	5600	7840	11200	13440	14560		

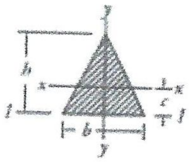
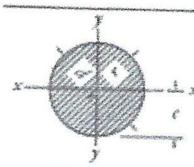
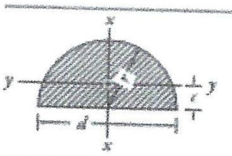
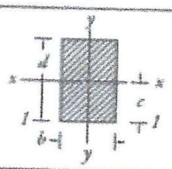
Using the above information and the table of results, produce (i) a load/extension graph over the complete test range, and (ii) a load/ extension graph to an enlarged scale over the elastic range of the specimen.

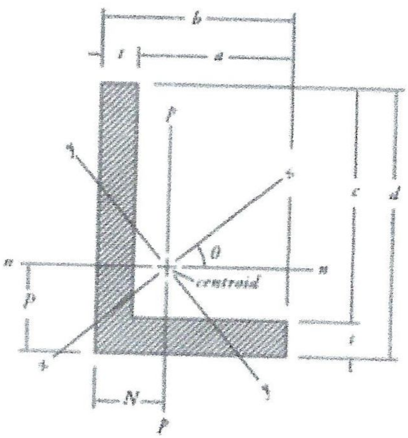
Using the two graphs and other information supplied, determine the values of;

(a) Young's modulus of elasticity;
(b) The ultimate tensile stress;
(c) The stress at the upper and lower yield points;
(d) The percentage reduction of area;
(e) The percentage elongation;
(f) The nominal and actual stress at fracture.

END OF EXAMINATION!!!! ALL THE BEST!!!!!!

Useful Properties of Areas

Section	Area	Centroidal distance	Second moment of inertia		Elastic section modulus		
		c	I_x, I_1	I_y	Z_x	Z_y	r_x
	$\frac{bh}{2}$	$\frac{h}{3}$	$I_x = \frac{bh^3}{36}$ $I_1 = \frac{bh^3}{12}$	$\frac{hb^3}{48}$	Apex $\frac{bh^2}{24}$ Base $\frac{bh^2}{12}$	$\frac{bh^2}{24}$	$4.24h$
	$3.14r^2$ $(= 0.785d^2)$	$r\left(= \frac{d}{2}\right)$	$0.785r^4$ $(= 0.0491d^4)$	$0.785r^4$	$0.785r^3$ $(= 0.0982d^3)$	$0.785r^3$	$0.5r$ $(= 0.25d)$
	$1.57r^2$	$0.424r$	$0.393r^4$	$0.110r^4$	$0.393r^3$	Crown $0.191r^3$ Base $0.259r^3$	$0.264r$
	bd	$\frac{d}{2}$	$I_x = \frac{bd^3}{12}$ $I_1 = \frac{bd^3}{3}$	$\frac{db^3}{12}$	$\frac{bd^2}{6}$	$\frac{db^2}{6}$	$0.289d$



x- and y-axis are the major and minor principal axis respectively (with $I_{xy} = 0$). Minimum $I = I_y$. The product second moment of inertia about the n-, p-axis (I_{np}) is −ve when the heel of the angle (with respect to the centroid) is in the 1st (top right) or 3rd (bottom left) quadrants and positive otherwise.

$$\tan 2\theta = \frac{2I_{np}}{I_n - I_p} \qquad A = t(b + c)$$

$$N = \frac{b^2 + ct}{2(b + c)} \qquad P = \frac{d^2 + at}{2(b + c)}$$

$$I_{np} = \pm \frac{abcdt}{4(b + c)}$$

$$I_n = \frac{1}{3}\left| t(d - P)^3 + bP^3 - a(P - t)^3 \right|$$

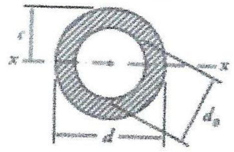
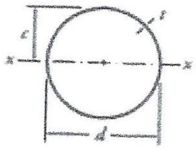
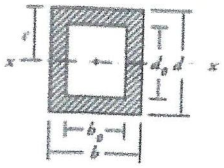
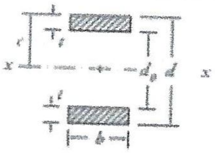
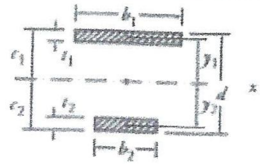
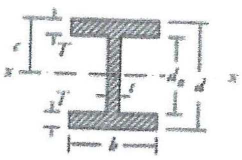
$$I_p = \frac{1}{3}\left| t(b - N)^3 + dN^3 - c(N - t)^3 \right|$$

$$I_x = \frac{I_n + I_p}{2} + \frac{I_n - I_p}{2\cos 2\theta}$$

$$I_y = \frac{I_n + I_p}{2} - \frac{I_n - I_p}{2\cos 2\theta}$$

CE 211 FINAL EXAMINATION

FORMULA SHEET

Section	Area A and Centroidal dist. A, c	Second moment of area I_x	Radius of gyration r_x	Elastic section modulus Z_x
	$A = \frac{\pi}{4}(d^2 - d_0^2)$ $c = \frac{d}{2}$	$I_x = \frac{\pi}{64}(d^4 - d_0^4)$	$r_x = \frac{1}{4}\sqrt{(d^2 + d_0^2)}$	$Z = \frac{\pi}{32d}(d^4 - d_0^4)$
	$A = \pi(d - t)t$ $c = \frac{d}{2}$	$I_x = \frac{\pi}{8}(d - t)^3 t$ $= 0.393(d - t)^3 t$	$r_x = 0.354d$	$Z = \frac{\pi}{4}\frac{(d - t)^3 t}{d}$
	$A = bd - b_0 d_0$ $c = \frac{d}{2}$	$I_x = \frac{1}{12}(bd^3 - b_0 d_0^3)$	$r_x = \sqrt{\frac{I_x}{A}}$	$Z_x = \frac{1}{6d}(bd^3 - b_0 d_0^3)$
	$A = 2bt$ $c = \frac{d}{2}$	$I_x = \frac{b}{12}(d^3 - d_0^3)$	$r_x = \sqrt{\frac{I_x}{A}}$	$Z_x = \frac{b}{6d}(d^3 - d_0^3)$
 $y_1 = \left(c_1 - \frac{t_1}{2}\right)$ $y_2 = \left(c_2 - \frac{t_2}{2}\right)$	$A = b_1 t_1 + b_2 t_2$ $c_1 = \frac{\frac{1}{2}b_1 t_1^2 + b_2 t_2\left(d - \frac{1}{2}t_2\right)}{A}$ $c_2 = d - c_1$	$I_x = \frac{b_1 t_1^3}{12} + \frac{b_2 t_2^3}{12} + b_1 t_1 y_1^2 + b_2 t_2 y_2^2$	$r_x = \sqrt{\frac{I_x}{A}}$	$Z_{top} = \frac{I_x}{c_1}$ $Z_{btm} = \frac{I_x}{c_2}$
	$A = 2bT + (d - 2T)t$ $c = \frac{d}{2}$	$d_0 = d - 2T$ $I_x = \frac{b}{12}(d^3 - d_0^3) + \frac{1}{12}d_0^3 t$	$r_x = \sqrt{\frac{I_x}{A}}$	$Z_x = \frac{I_x}{c}$

For a *solid or hollow shaft* of uniform circular cross-section throughout its length, the theory of pure torsion states that

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

where T is the applied external torque, constant over length L;

J is the polar second moment of area of shaft cross-section

$= \dfrac{\pi D^4}{32}$ for a solid shaft and $\dfrac{\pi(D^4 - d^4)}{32}$ for a hollow shaft;

D is the outside diameter; R is the outside radius;

d is the inside diameter;

τ is the shear stress at radius R and is the maximum value for both solid and hollow shafts;

G is the modulus of rigidity (shear modulus); and

θ is the angle of twist in *radians* on a length L.

For *very thin-walled hollow shafts*

$J = 2\pi r^3 t$, where r is the mean radius of the shaft wall and t is the thickness.

Shear stress and shear strain are related to the angle of twist thus:

$$\tau = \frac{G\theta}{L}R = G\gamma$$

Strain energy in torsion is given by

$$U = \frac{T^2 L}{2GJ} = \frac{GJ\theta^2}{2L}$$

For *solid shafts*

$$U = \frac{\tau^2}{4G} \times \text{volume}$$

For a circular shaft subjected to *combined bending and torsion* the *equivalent bending moment* is

$$M_e = \tfrac{1}{2}[M + \sqrt{(M^2 + T^2)}]$$

and the *equivalent torque* is

$$T_e = \tfrac{1}{2}\sqrt{(M^2 + T^2)}$$

where M and T are the applied bending moment and torque respectively.

The *shear stress* in a beam at any transverse cross-section in its length, and at a point a vertical distance y from the neutral axis, *resulting from bending* is given by

$$\tau = \frac{QA\bar{y}}{Ib} \quad \text{or} \quad \tau = \frac{Q}{Ib}\int y\,dA$$

where Q is the applied vertical shear force at that section; A is the area of cross-section "above" y, i.e. the area between y and the outside of the section, which may be above or below the neutral axis (N.A.); $\bar{y}$ is the distance of the centroid of area A from the N.A.; I is the second moment of area of the complete cross-section; and b is the breadth of the section at position y.

For **rectangular sections**,

$$\tau = \frac{6Q}{bd^3}\left[\frac{d^2}{4} - y^2\right] \quad \text{with} \quad \tau_{max} = \frac{3Q}{2bd} \quad \text{when} \quad y = 0$$

For **I-section beams** the *vertical shear* in the web is given by

$$\tau = \frac{Q}{2I}\left[\frac{h^2}{4} - y^2\right] + \frac{Qb}{2It}\left[\frac{d^2}{4} - \frac{h^2}{4}\right]$$

with a *maximum* value of

$$\tau_{max} = \frac{Qh^2}{8I} + \frac{Qb}{2It}\left[\frac{d^2}{4} - \frac{h^2}{4}\right]$$

The *maximum* value of the *horizontal shear* in the flanges is

$$\tau_{max} = \frac{Qb}{4I}(d - t_1)$$

For **circular sections**

$$\tau = \frac{4Q}{3\pi R^2}\left[1 - \left(\frac{y}{R}\right)^2\right]$$

with a *maximum* value of

$$\tau_{max} = \frac{4Q}{3\pi R^2}$$

The *shear centre* of a section is that point, in or outside the section, through which load must be applied to produce zero twist of the section. Should a section have two axes of symmetry, the point where they cross is automatically the shear centre.

The shear centre of a **channel section** is given by

$$e = \frac{k^2h^2t}{4I}$$