THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING - 2ND YEAR DEGREE

SECOND SEMESTER EXAMINATIONS - 2021

CE 222 – STRUCTURAL ANALYSIS EXAM

DATE: FRIDAY, 29TH OCTOBER 2021 – 08:20 A.M.

VENUE: STRUCTURES LECTURE THEATRE (SLT)

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. You have 10 minutes to read the paper before the examination starts. You must **not** begin writing during this time.
2. **There are Eight (8) Questions in this paper. Answer any FIVE (5) of the eight questions.**
3. Use only ink. Do not use pencils for writing except for drawings and sketches.
4. Start each question on a new page and show all your calculations in the answer book provided. No other material will be accepted.
5. Write on **one side of the page only** and keep the margins clear.
6. **Write your NAME and Student NUMBER clearly on the front page. Do it now.**
7. Calculators are permitted in the examination. Note and textbooks are not allowed.
8. Marks for each of the question are given within parenthesis at the end of each question.
9. **Switch your mobile phone OFF**.

Question One **[Integration Method]**

A simply supported beam shown in **Figure Q1A** is of span 20 m and carries two concentrated loads **4 KN** at 8 m and **10 kN** at 12 m from one end.

Calculate;

(a) the deflection under each load, **(6 marks)**

(b) the maximum deflection. **(4 marks)**

E = 200, 000 N/mm^2, I = 10^9 mm^4

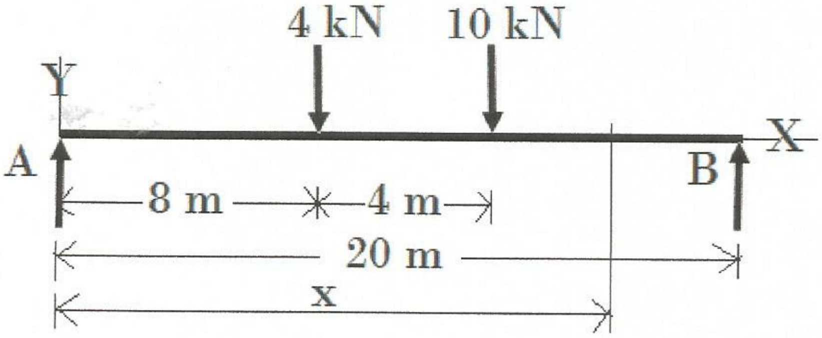


Figure Q1A

(c) Determine the equation for the elastic curve for the cantilevered beam in **Figure Q1B**. EI is constant. **(10 marks)**

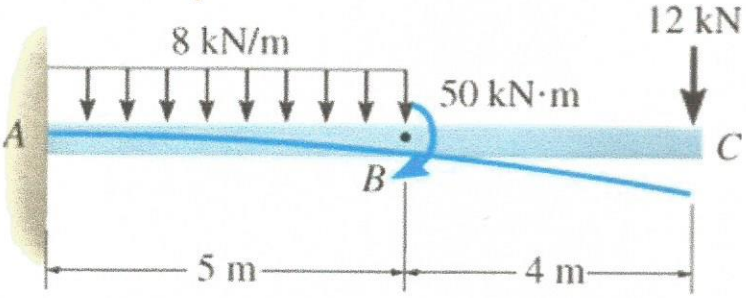


Figure Q1B

Question Two **[Moment-Area Method]**

Using the **Moment-Area Method,** determine the slopes at **B** and **C** for the mild steel beam shown in **Figure Q2**, and also the deflection at the load point **C**. **(20 marks)**

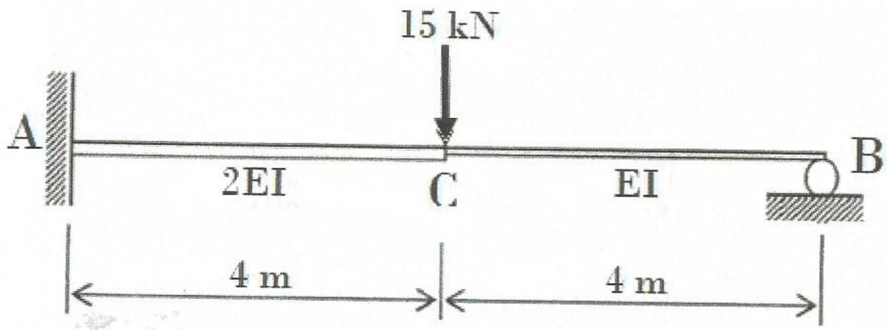


Figure Q2

Question Three **[Slope Deflection Method – Beam]**

Use the ***Slope Deflection Method*** to determine the moments at A, B, C and D for the steel beam shown in **Figure Q3** below and plot the shear force and bending moment diagrams for the beam. Assume **EI** is constant. **(20 marks)**

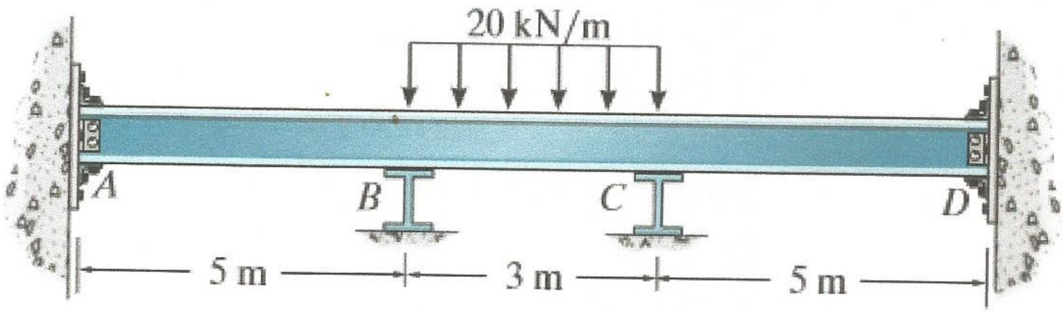


Figure Q3

Question Four **[Energy Methods: Virtual Work/Castigliano's Theorem]**

(a) Determine the vertical displacement at joint **A** for the truss shown in **Figure Q4**. Each bar is made of steel and has a cross-sectional area of **600 mm²**.
Take **E = 200 GPa**. Use the **Method of Virtual Work**. EI is constant. **(10 marks)**

(b) Solve (a) using **Castigliano's theorem.** **(10 marks)**

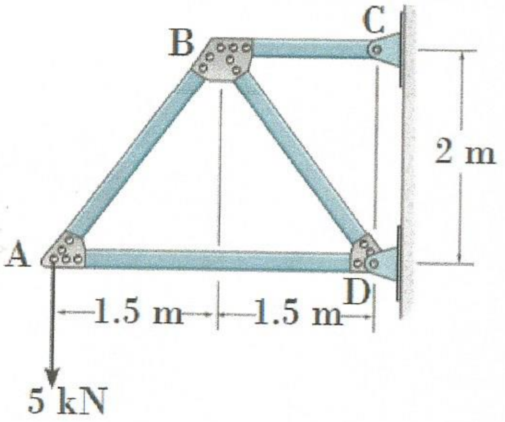


Figure Q4

Question Five **Stiffness Matrix Method – Beam**

Determine the reactions at the supports shown in **Figure Q5**. Assume 1 and 3 are fixed and 2 is a roller. **EI** is constant.
(20 marks)

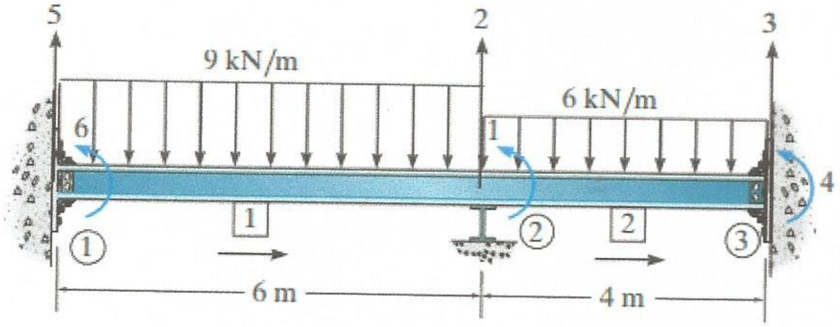


Figure Q5

Question Six **Moment-Area/ Conjugate Beam Method**

(a) Determine the slope and the displacement at the end **C** of the beam in **Figure Q6** using the Moment-Area Method. **E = 200 GPa, I = 70x10^6 mm^4.** **(10 marks)**

(b) Solve the problem in (a) using the conjugate-beam method. **(10 marks)**

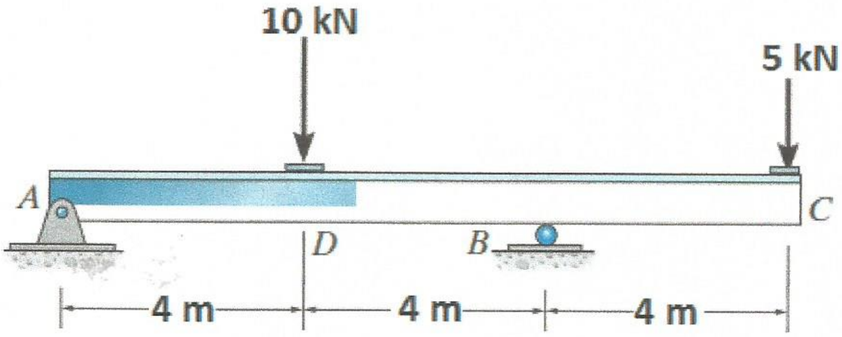


Figure Q6

Question Seven **Slope Deflection Method - Frame**

Using the Slope Deflection Method, determine the reactions at the supports and the moment at **B** for the frame shown in Figure Q7, then draw the moment diagram for each member of the frame. Assume the support at **A** is fixed and **C** is pinned.

EI is constant. **(20 marks)**

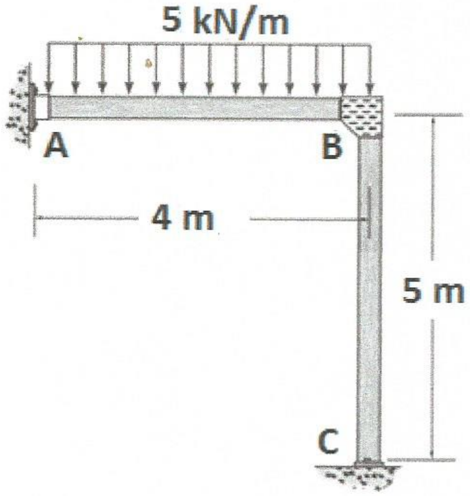


Figure Q7

Question Eight **[Energy Methods: Virtual Work/Castigliano's Theorem]**

(a) Determine the slope and displacement at point **A** for the beam shown in **Figure Q8**. Use the principle of virtual work. **EI** is constant. **(10 marks)**

(b) Solve the problem in (a) using Castigliano's theorem. **(10 marks)**

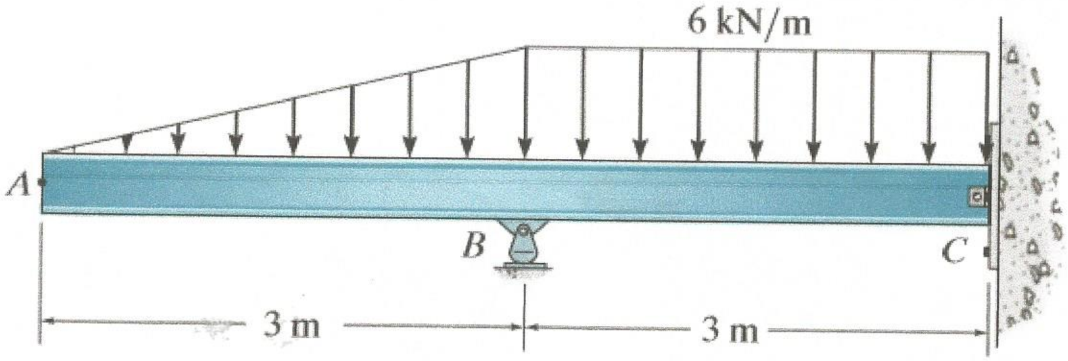
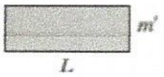
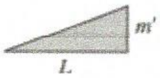
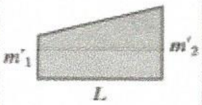
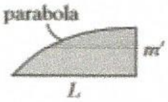
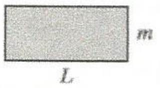
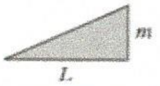
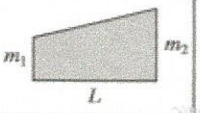
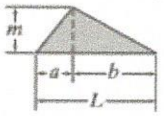
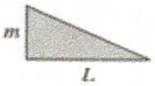


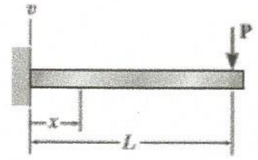
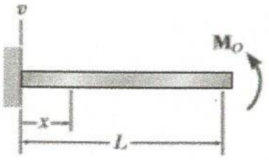
Figure Q8

END OF EXAMINATION! ALL THE BEST!

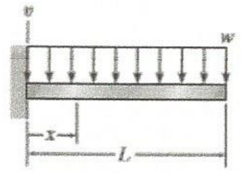
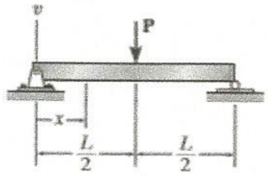
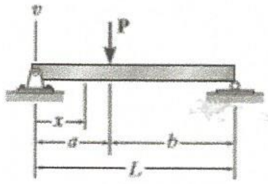
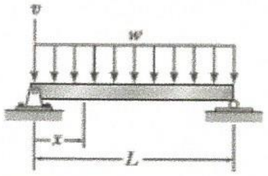
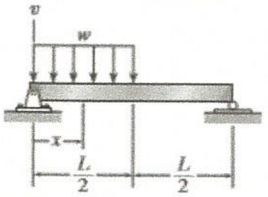
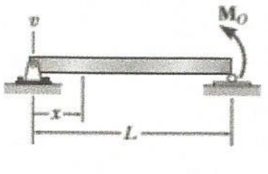
Table for Evaluating $\int_0^L m\,m'\,dx$

$\int_0^L m\,m'\,dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m'_1+m'_2)L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m'_1+2m'_2)L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1+m_2)L$	$\frac{1}{6}m'(m_1+2m_2)L$	$\frac{1}{6}[m'_1(2m_1+m_2)+m'_2(m_1+2m_2)]L$	$\frac{1}{12}[m'(3m_1+5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L+a)$	$\frac{1}{6}m[m'_1(L+b)+m'_2(L+a)]$	$\frac{1}{12}mm'\left(3+\frac{3a}{L}-\frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m'_1+m'_2)L$	$\frac{1}{4}mm'L$

Beam Deflections and Slopes

Loading	$v+\uparrow$	$\theta+$ ↶	Equation $+\uparrow$ + ↶
	$v_{max}=-\frac{PL^3}{3EI}$ at $x=L$	$\theta_{max}=-\frac{PL^2}{2EI}$ at $x=L$	$v=\frac{P}{6EI}(x^3-3Lx^2)$
	$v_{max}=\frac{M_OL^2}{2EI}$ at $x=L$	$\theta_{max}=\frac{M_OL}{EI}$ at $x=L$	$v=\frac{M_O}{2EI}x^2$

Beam Deflections and Slopes (continued)

	$v_{max} = -\dfrac{wL^4}{8EI}$ at $x = L$	$\theta_{max} = -\dfrac{wL^3}{6EI}$ at $x = L$	$v = -\dfrac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
	$v_{max} = -\dfrac{PL^3}{48EI}$ at $x = L/2$	$\theta_{max} = \pm\dfrac{PL^2}{16EI}$ at $x = 0$ or $x = L$	$v = -\dfrac{P}{48EI}(4x^3 - 3L^2x),$ $0 \le x \le L/2$
		$\theta_L = -\dfrac{Pab(L + b)}{6LEI}$ $\theta_R = \dfrac{Pab(L + a)}{6LEI}$	$v = -\dfrac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$v_{max} = -\dfrac{5wL^4}{384EI}$ at $x = \dfrac{L}{2}$	$\theta_{max} = \pm\dfrac{wL^3}{24EI}$	$v = -\dfrac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$
		$\theta_L = -\dfrac{3wL^3}{128EI}$ $\theta_R = \dfrac{7wL^3}{384EI}$	$v = -\dfrac{wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = -\dfrac{wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$v_{max} = -\dfrac{M_0L^2}{9\sqrt{3}EI}$	$\theta_L = -\dfrac{M_0L}{6EI}$ $\theta_R = \dfrac{M_0L}{3EI}$	$v = -\dfrac{M_0x}{6EIL}(L^2 - x^2)$

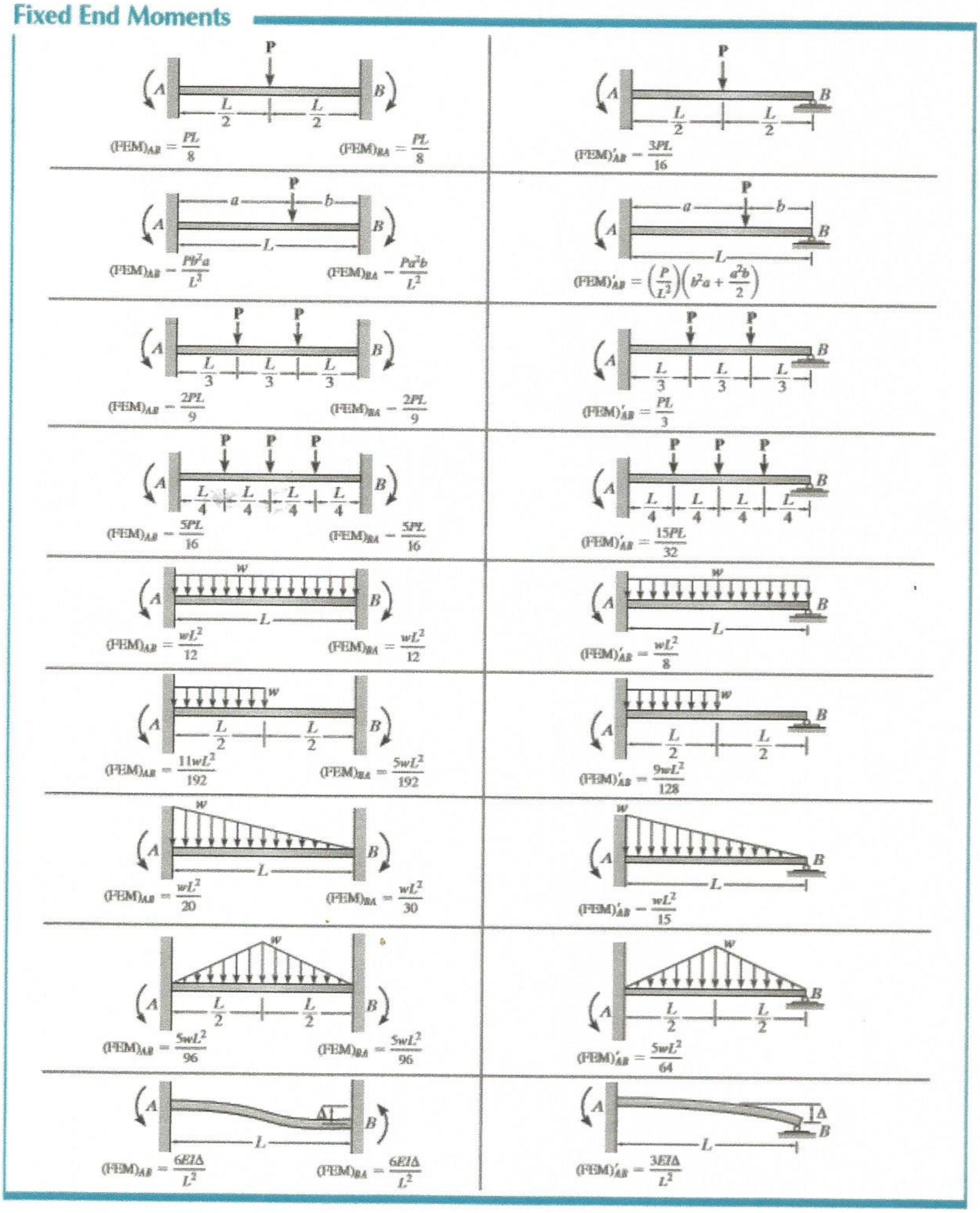
$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For Internal Span or End Span with Far End Fixed

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

Only for End Span with Far End Pinned or Roller Supported

Fixed End Moments



Member Global Stiffness Matrix

$$\boxed{\mathbf{q} = \mathbf{k}'\mathbf{T}\mathbf{D}}$$

$$\mathbf{Q} = \mathbf{T}^T\mathbf{k}'\mathbf{T}\mathbf{D}$$

$$\boxed{\mathbf{Q} = \mathbf{k}\mathbf{D}}$$

$$\boxed{\mathbf{k} = \mathbf{T}^T\mathbf{k}'\mathbf{T}}$$

$$\mathbf{k} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \end{bmatrix} \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}$$

$$\mathbf{k} = \frac{AE}{L} \begin{array}{c} \begin{array}{cccc} N_x & N_y & F_x & F_y \end{array} \\ \begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \end{array} \begin{array}{c} N_x \\ N_y \\ F_x \\ F_y \end{array}$$

Beam Analysis Using Stiffness Method

$$
\begin{bmatrix} q_{Ny'} \\ q_{Nz'} \\ q_{Fy'} \\ q_{Fz'} \end{bmatrix}
=
\begin{array}{c}
\begin{array}{cccc} \quad N_{y'} & \qquad N_{z'} & \qquad F_{y'} & \qquad F_{z'} \end{array} \\
\begin{bmatrix}
\frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
\frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
\frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\end{array}
\begin{bmatrix} d_{Ny'} \\ d_{Nz'} \\ d_{Fy'} \\ d_{Fz'} \end{bmatrix}
$$

These equations can also be written in abbreviated form as

$$\mathbf{q} = \mathbf{kd}$$

Structure Stiffness Equation

$$\mathbf{Q} = \mathbf{KD}$$

Q and **D** are column matrices that represent both the known and unknown loads and displacements. Partitioning the stiffness matrix into the known and unknown elements of load and displacement, we have

$$\begin{bmatrix} \mathbf{Q}_k \\ \hline \mathbf{Q}_u \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \hline \mathbf{K}_{21} & \mathbf{K}_{22} \end{array}\right] \begin{bmatrix} \mathbf{D}_u \\ \hline \mathbf{D}_k \end{bmatrix}$$

which when expanded yields the two equations

$$\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$$

$$\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$$