THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS – 2022

FIRST YEAR BACHELOR IN COMPUTERSCIENCE

CS122 – MATHEMATICS FOR COMPUTER SCIENCE

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklet/s.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains five (5) questions. You should attempt all the questions.
4. Make sure you have 5 pages.
5. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
6. Start the answer for each question on a new page.
7. Do not use red ink or pencil.
8. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
9. Scientific and business calculators are allowed in the examination room.
10. The last two pages contains a formula sheet for students information.

MARKING SCHEME

Marks are indicated at the beginning of each question. Total mark is 100.

Question 1 (5 + 5 +15 = 25 Marks)

(a) If $A = \begin{bmatrix} 6 & 0 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$, find matrix C, if C = A.B.

(b) Find the value of determinant of matrix A if A = $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$.

(c) Solve the following system of linear equations by Gauss – Jordan Elimination method.

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

Question 2 (5 + 5 + 10 = 20 Marks)

(a) Find the unit vector of $\bar{r} = 5\hat{\imath} + 6\hat{\jmath} - \hat{k}$.

(b) Find the value of given cross product $(2\hat{\imath} - \hat{\jmath} + \hat{k})$ **x** $(\hat{\imath} + \hat{\jmath} + 2\hat{k})$.

(c) Find the value of m so that $(2\hat{\imath} + \hat{\jmath} - \hat{k})$, $(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $(3\hat{\imath} + 5\hat{\jmath} - m\hat{k})$ are coplanar.

Question 3 (5 + 5 + 5 = 15 Marks)

(a) Find the Truth tables for any proposition p and q, $E = \neg P \wedge (\neg Q \vee R)$.

(b) Show that the propositions $\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ are logically equivalent.

(c) Let proposition P be "It is cold" and let Q be "It is raining". Give a simple verbal sentence which describes each of the following statements:

(i) $\neg P$ (ii) $P \wedge Q$ (iii) $\neg P \wedge Q$.

Question 4 (5 + 5 + 10 = 20 Marks)

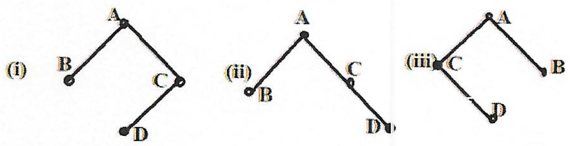
(a) How many different words can be formed from the alphabets of "PAPUAGUINEA".

(b) Eight subject tests are to be taken in a particular week. Show by Pigeon Hole Principle at least two tests will be conducted on the same day.

(c) Use Mathematical Induction to show that $1 + 2 + 2^2 + \ldots\ldots + 2^n = 2^{n+1} - 1$ for all non-negative integer n.

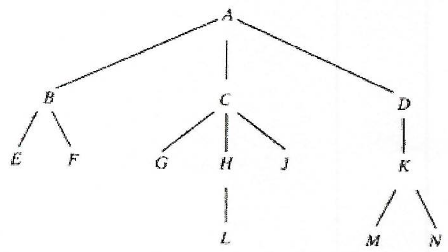
Question 5 (5 + 5 + 10 = 20 Marks)

(a) Find the number of Edges and Nodes in the following trees



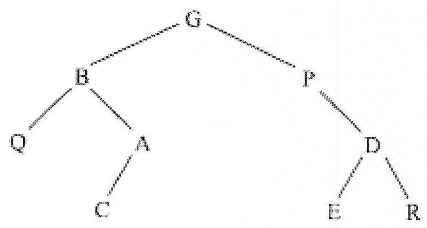
(b) For the following tree find out:

i. Number of children C and F have
ii. Nodes having one child each, two children and three children.
iii. Which node have no children.



(c) For the given tree find the following traversal.

i. Inorder ii. Preorder iii. Postorder



END OF EXAMINATION

Reference Material

Multiplication of two matrices

Two matrices can be multiplied together only when the number of columns in the first is equal to the number of rows in the second.

$$\text{e.g. if } \mathbf{A} = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \text{ and } \mathbf{b} = (b_i) = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\text{then } \mathbf{A}.\mathbf{b} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_1 + a_{12}b_2 + a_{13}b_3 \\ a_{21}b_1 + a_{22}b_2 + a_{23}b_3 \end{pmatrix}$$

Determinant

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$ Expanding |A| along C_1, we get

$$|A| = a_{11}(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}a_{22})$$

Elementary Transformations: When we apply elementary operation then value of matrix will not changes. There are three type of Elementary Operations/Transformations

1. R_{ij} or C_{ij} : Two Rows/Column may be switched.
2. aR_i or aC_i: Any Row/Column may be multiplied by a number.
3. $R_i \rightarrow aR_j + R_i$ or $C_i \rightarrow aC_j + C_i$: At any time replace a row with a multiple of **another row** added to the row.

Echelon Form

Echelon form. Row reduced form or Upper triangular form

Procedure: Apply Elementary transformation and increase zero on left hand side of the matrix when we move towards down

Gauss Jordan method: Method to find out value of variable in system of equation

$a_1 x + b_1 y + c_1 z = d_1$
$a_2 x + b_2 y + c_2 z = d_2$
$a_3 x + b_3 y + c_3 z = d_3$

System of equation converted in Augmented matrix

$$(\mathbf{A}\,|\,\mathbf{B}) = \begin{pmatrix} a_1 & b_1 & c_1 & & d_1 \\ a_2 & b_2 & c_2 & | & d_2 \\ a_3 & b_3 & c_3 & & d_3 \end{pmatrix}$$

Apply elementary transformation and convert Augmented matric in Normal form

$$(\mathbf{A}\,|\,\mathbf{B}) = \begin{pmatrix} 1 & 0 & 0 & & p \\ 0 & 1 & 0 & | & r \\ 0 & 0 & 1 & & m \end{pmatrix}$$

Gives the value x = p, y = r and z = m

Product of vectors

Dot product: $(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) \bullet (p\hat{\imath} + q\hat{\jmath} + r\hat{k}) = a \times p\,(\hat{\imath} \bullet \hat{\imath}) + b \times q\,(\hat{\jmath} \bullet \hat{\jmath}) + c \times r\,(\hat{k} \bullet \hat{k})$ and $(\hat{\imath} \bullet \hat{\imath}) = (\hat{\jmath} \bullet \hat{\jmath}) = (\hat{k} \bullet \hat{k}) = 1$

Cross products: $(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) \times (p\hat{\imath} + q\hat{\jmath} + r\hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a & b & c \\ p & q & r \end{vmatrix}$

Coplanar vector: Three vectors $\overline{r_1} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$, $\overline{r_2} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$, $\overline{r_3} = a_3\hat{\imath} + b_3\hat{\jmath} + c_3\hat{k}$, are coplanar if volume made by them is zero i.e. $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

Truth Tables

P	Q	$\neg P$	$P \vee Q$	$P \wedge Q$	$P \oplus Q$
T	T	F	T	T	F
T	F	F	T	F	T
F	T	T	T	F	T
F	F	T	F	F	F

Permutations with Repetitions $P(n;\, n_1, n_2, \ldots, n_r) = \dfrac{n!}{n_1!\,n_2!\ldots n_r!}$

number of permutations of n objects of which n_1 are alike, n_2 are alike, . . ., n_r are alike.

Pigeonhole Principle:

If n pigeonholes are occupied by $n + 1$ or more pigeons, then at least one pigeonhole is occupied by more than one pigeon.

Principle of Mathematical Induction

Let P be a proposition defined on the positive integers **N**; that is, P(n) is either true or false for each $n \in$ **N**. P satisfy following:

(i) **Basic Step:** P (1) is true.
(ii) **Inductive Step:** P (k + 1) is true whenever P (k) is true.

Tree Traversal

Moving from one node to other node of a tree is known as Tree Traversal

Preorder: NLR- Node-Left-Right
Inorder: LNR- Left-Node-Right
Postorder: LRN- Left-Right-Node