THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS - 2022

THIRD YEAR BACHELORS IN COMPUTER SCIENCE

CS313–NUMERICAL METHODS

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. There are **five** questions, you should answer all questions.
4. All answers must be written in examination booklets only. No other written material will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes and textbooks are not allowed in the examination room. All mobile phones and electronic/recording devices must be switched off during the examination period.
7. Scientific and business calculators are allowed in the examination room.
8. A formula and information sheet is attached.

MARKING SCHEME

Marks are indicated at the beginning of each question. They total 100.

QUESTION 1 **[11 + 3 + 6 = 20 Marks]**

Let $f(x)=\ln(\sqrt{4x+1})$. Answer the following questions accordingly. (Leave the final answers correct to 4 decimal places where necessary.)

(a) Find the third Taylor polynomial $P_3(x)$ for $f(x)$ about $a=0$.

(b) Apply $P_3(0.2)$ to approximate $f(0.2)$.

(c) Find the upper bound error $R_3(0.2)$ and compare it the actual error $E_3(0.2)$ using the error formula.

QUESTION 2 **[6 + 6 + 8 = 20 Marks]**

(a) Find all values of k that makes the given matrix singular.

$$\mathbf{A}=\begin{bmatrix}3 & 1 & k\\ 1 & 4 & 3\\ 0 & k & -2\end{bmatrix}$$

(b) Partition the two matrices **A** and **B** given below in such a way for faster computing **AB.** The actual computation of **AB** is not required.

$$\mathbf{A}=\begin{bmatrix}2 & 1 & 3 & 4 & 1\\ 5 & 3 & -1 & 2 & 4\\ -4 & 2 & -1 & 3 & 2\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}1 & 3 & 1 & 0\\ 2 & 2 & -2 & 3\\ 3 & 5 & -3 & 1\\ 4 & 4 & 1 & 4\\ 5 & 1 & 2 & 5\end{bmatrix}$$

(c) Find the approximate solutions to the following linear system of equations using the Gauss – Seidel method. Only the first two (2) iterations are required. (Round off the final answers correct to 4 decimal places.)

$$\begin{cases}3x & -2y & & +w & =5\\ x & +2y & +z & -3w & =4\\ -2x & +3y & +2z & +2w & =3\\ 3x & & -z & +2w & =2\end{cases}$$

QUESTION 3 **[12 + 8 = 20 Marks]**

Apply the indicated numerical integration methods to evaluate the given integral.(Leave the final answer correct to 2 decimal places.)

$$\int_0^4 \frac{1}{\sqrt{4x^2+9}}\,dx;\quad n=4.$$

(a) Use the Trapezoidal method to evaluate the integral.

(b) Use the Simpson's $\frac{1}{3}$ rule to evaluate the integral.

QUESTION 4 **[10 + 10 = 20 Marks]**

(a) Solve the following initial value problem for y when $x = 3$ using the Euler method with $h = 1.5$. (Leave the final answer correct to 2 decimal places.)

$$\frac{dy}{dx} = 3e^{-2x} - 0.2y; \quad y(0) = 5$$

(b) Repeat part (a) using the Runge – Kutta 4[th] order method with $h = 3$. (Leave the final answer correct to 2 decimal places.)

QUESTION 5 **[12 + 8 = 20 Marks]**

(a) Find the eigenvalues and their corresponding eigenvectors for the given matrix.

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

(b) For the 3 x 3 matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{pmatrix}$, its eigenvalues are: $\lambda_1 = 1, \ \lambda_2 = 2$ & $\lambda_3 = 3$. Find the eigenvector corresponding to $\lambda_2 = 2$.

END OF EXAMINATION

FORMULA/INFORMATION SHEET

1. $P_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots + \frac{f^n(0)x^n}{n!}$

2. $P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots + \frac{f^n(a)(x-a)^n}{n!}$

3. Error in $T_n \le \frac{M(b-a)^3}{12n^2}$; $M = Max\ of\ |f''(x)|$ over $[a,b]$.

4. Error in $S_n \le \frac{M(b-a)^5}{180n^4}$; $M = Max\ of\ |f^{(4)}(x)|$ over $[a,b]$.

4. $x_i^{(k)} = \frac{1}{a_{ii}}\left[\sum_{j=1, j\ne i}^{n}\left(-a_{ij}x^{(k-1)}\right) + b_i\right]$

5 $x_i^{(k)} = \frac{1}{a_{ii}}\left[-\sum_{j=1}^{i-1}\left(a_{ij}x^{(k)}\right) - \sum_{j=i+1}^{n}\left(a_{ij}x^{(k-1)}\right) + b_i\right]$

6. $\int_a^b f(x)dx = \frac{h}{2}[f_0 + 2f_1 + 2f_2 + \ldots.. + 2f_{n-1} + f_n]$; $h = \frac{b-a}{n}$

7. $\int_a^b f(x)dx = \frac{h}{3}[f_0 + 4f_1 + 2f_2 + \ldots.. + 4f_{n-1} + f_n]$; $h = \frac{b-a}{n}$

8. $\int_a^b f(x)dx = \frac{3h}{8}[f_0 + 3f_1 + 3f_2 + 2f_3 + 3f4 + 3f5 + 2f6 + \ldots.. + f_n]$; $h = \frac{b-a}{n}$

9. $\frac{dy}{dx} = f(x,y); \quad y(x_o) = y_0.$

(a) $y_{n+1} = y_n + hf(x_n, y_n)$, h will be given.

(b) $y_{n+1} = y_n + \frac{h}{6}[k_1 + 2k_2 + 2k_3 + k_4]$, h will be given. Here,

$k_1 = f(x_n, y_n)$;
$k_2 = f(x_n + 0.5h, y_n + 0.5hk_1)$;
$k_3 = f(x_n + 0.5h, y_n + 0.5hk_2)$;
$k_4 = f(x_n + h, y_n + hk_3)$.

10. $\mathbf{Ax} = \lambda\mathbf{x}$
$(\mathbf{A} - \mathbf{I}\lambda)\mathbf{x} = \mathbf{0}$
$|\mathbf{A} - \mathbf{I}\lambda| = 0$