THE PAPUA NEW UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS, 2023

THIRD YEAR BACHELORS OF SCIENCE COMPUTER SCIENCE

# CS313 NUMERICAL METHODS

TIME ALLOWED: THREE (3) HOURS

## INSTRUCTIONS TO CANDIDATES

1. You have 10 minutes to read this paper. You **must not** begin writing during this time.
2. There are **five(5)** questions in this examination paper. Answer **all** questions.
3. Write all answers in the answer booklet provided.
4. All working should be shown clearly on the answer booklets.
5. All rough work should be **crossed out** leaving the final answer for clarity.
6. Start each question on a new page and clearly write its question number at the top of the page.
7. Calculators are allowed in the examination room.
8. Write your name and id number clearly on the examination booklets.
9. Mobile phones and other recording devices must be **switched off** during the examination period.
10. The last page is the formula and information sheet.

**MARKING SCHEME**
Marks are indicated at the beginning of each question. The total mark is **100 marks.**

**QUESTION 1 [10 + 3+ 7 = 20 Marks]**

Let $f(x)=\ln\left(\frac{1}{x^3}\right)$. Answer the following questions accordingly. *(Answers correct to 4 decimal places.)*

(a) Find the third Taylor polynomial $P_3(x)$ for $f(x)=\ln\left(\frac{1}{x^3}\right)$ about $a=1$.

(b) Apply $P_3(0.99)$ to approximate $f(0.99)$.

(c) Find an upper bound for error $R_3(0.99)$ and compare it with the actual error $E_3(0.99)$ using the error formula.

**QUESTION 2 [5 + 7 + 8 = 20 Marks]**

(a) Evaluate the determinant of the given matrix.

$$\mathbf{A}=\begin{bmatrix} 4 & -5 & 0 & 4 & 5 \\ 0 & 3 & 0 & 0 & 0 \\ -1 & 2 & 5 & 3 & -4 \\ 5 & -2 & 0 & 3 & 0 \\ 3 & 5 & 0 & 2 & 0 \end{bmatrix}$$

(b) Solve the following system of equations using the inverse matrix method.

$$\begin{aligned} 3x-2y+5z &= -10 \\ 5x+y-2z &= 17 \\ -2x+3y+z &= 3 \end{aligned}$$

Note that the following information refer to the above equations.

$$\mathbf{A}=\begin{bmatrix} 3 & -2 & 5 \\ 5 & 1 & -2 \\ -2 & 3 & 1 \end{bmatrix}, \mathbf{b}=\begin{bmatrix} -10 \\ 17 \\ 3 \end{bmatrix}, \ \mathbf{X}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, \mathbf{A}^{-1}=\frac{1}{108}\begin{bmatrix} 7 & 17 & -1 \\ -1 & 13 & 31 \\ 17 & -5 & 13 \end{bmatrix}$$

(c) Find the first **two iterations** of Gauss-Seidel method for the following linear system using $X^{(0)}=0$. *(Answer correct to 4 decimal places.)*

$$\begin{cases} 11x + 2w = 7 \\ x - 25y + z = 6.5 \\ 2y + 20z - w = 8 \\ 3z + 15w = 10 \end{cases}$$

## QUESTION 3 [7 + 6 + 7 = 20 Marks]

Apply the indicated numerical integration methods to evaluate the given integral. *(Answer correct to 2 decimal places.)*

$$\int_0^4 \ln(2x+1)dx; \quad n = 4.$$

(a) Use the Trapezoidal method to evaluate the integral.

(b) Use the Simpson's $\frac{1}{3}$ rule to evaluate the integral.

(c) Find the value of n (the number of subintervals) that would estimate part (a) of error less than or equal to 0.01.

## QUESTION 4 [10 + 10 = 20 Marks]

(a) Solve the following initial value problem for $y$ when $x = 3$ using the Euler method with, $h = 1$. *(Answer correct to 4 decimal places.)*

$$\frac{dy}{dx} = 4xe^{-0.5x} - 0.5y; \quad y(0) = 5$$

(b) Repeat part (a) using the Runge – Kutta 4th order method with $h = 1.5$. *(Answer correct to 4 decimal places.)*

## QUESTION 5 [12 + 8 = 20 Marks]

Study the two matrices below.

(i) $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$ (ii) $\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{pmatrix}$

(a) Find the eigenvalues and their corresponding eigenvectors for the 2 by 2 matrix in part (i).

(b) The eigenvalues for the 3 x 3 matrix in part (ii) are: $\lambda_1 = 1$, $\lambda_2 = 2$ & $\lambda_3 = 3$.
Find the eigenvector corresponding to $\lambda_2 = 2$.

**END OF EXAMINATION**

# FORMULA/INFORMATION SHEET

1. $P_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots + \frac{f^n(0)x^n}{n!}$

2. $P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots + \frac{f^n(a)(x-a)^n}{n!}$

3. Error in $T_n \leq \frac{M(b-a)^3}{12n^2}$; $M = Max\ of\ |f''(x)|$ over $[a,b]$.

4. Error in $S_n \leq \frac{M(b-a)^5}{180n^4}$; $M = Max\ of\ |f^{(4)}(x)|$ over $[a,b]$.

5. $E_n(x) = |f(x) - P_n(x)| \leq R_n(x) = \frac{f^{(n+1)}(\varsigma(x))}{(n+1)!}(x-a)^{n+1}$

6. $x_i^{(k)} = \frac{1}{a_{ii}}\left[\sum_{j=1, j\neq i}^{n}\left(-a_{ij}x^{(k-1)}\right) + b_i\right]$

7. $x_i^{(k)} = \frac{1}{a_{ii}}\left[-\sum_{j=1}^{i-1}\left(a_{ij}x^{(k)}\right) - \sum_{j=i+1}^{n}\left(a_{ij}x^{(k-1)}\right) + b_i\right]$

8. $\int_a^b f(x)dx = \frac{h}{2}[f_0 + 2f_1 + 2f_2 + \ldots + 2f_{n-1} + f_n]$; $h = \frac{b-a}{n}$

9. $\int_a^b f(x)dx = \frac{h}{3}[f_0 + 4f_1 + 2f_2 + \ldots + 4f_{n-1} + f_n]$; $h = \frac{b-a}{n}$

10. $\int_a^b f(x)dx = \frac{3h}{8}[f_0 + 3f_1 + 3f_2 + 2f_3 + 3f4 + 3f5 + 2f6 + \ldots + f_n]$; $h = \frac{b-a}{n}$

11. $\frac{dy}{dx} = f(x,y)$; $y(x_o) = y_0$.

    (a) $y_{n+1} = y_n + hf(x_n, y_n)$, $h$ will be given.

    (b) $y_{n+1} = y_n + \frac{h}{6}[k_1 + 2k_2 + 2k_3 + k_4]$, $h$ will be given. Here,

    $k_1 = f(x_n, y_n)$;
    $k_2 = f(x_n + 0.5h, y_n + 0.5hk_1)$;
    $k_3 = f(x_n + 0.5h, y_n + 0.5hk_2)$;
    $k_4 = f(x_n + h, y_n + hk_3)$.

12. $\mathbf{Ax} = \lambda\mathbf{x}$
    $(\mathbf{A} - \mathbf{I}\lambda)\mathbf{x} = \mathbf{0}$
    $|\mathbf{A} - \mathbf{I}\lambda| = 0$

13. $\mathbf{Ax} = \mathbf{b}$ or $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$

14. $\frac{d}{dx}(\ln(ax+b)) = \frac{a}{ax+b}$

15. $\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$