THE PAPUA NEW GUINEA
UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS 2021

FOURTH YEAR COMPUTER SCIENCE

CS405 - LINEAR PROGRAMMING

TIME ALLOWED – 3 HOURS

Information for Candidates

1. Write your name, student number, and program of study clearly on the front page of your answer booklet. Do it **now**.
2. You have 10 minutes to read this examination paper. During this time you must **NOT** write **inside** your answer booklet. You can make notes on the examination paper.
3. Scientific calculators are permitted. Other electronic devices are not permitted. Notes and headphones are not permitted.
4. Start each question on a new page. After you have finished the exam, indicate the order in which you answered questions in the left column of the marks box on the cover of the answer booklet.
5. There are 5 questions. You should attempt all questions. Question 5 has options – select one.
6. Total marks is 90. Marks for questions and question parts are shown at the top of each question.

QUESTION 1 [3 + 2 + 1 + 1 + 3 + 3 + 3 + 4 = 20 marks]

This question involves linear programs (LPs) that are in phase II form.

(a) Write down an LP with three variables and 2 constraints (and an objective function) that could be solved using Simplex phase II.

(b) Why MUST your problem (and any problem that can be formulated into phase II form) have a feasible solution?

(c) Convert the constraint $x_1 - x_2 + 3x_3 \geq -2$ so that it can be used with the Simplex algorithm:

The remaining parts to this question involve this augmented matrix:

x_1	x_2	x_3	s_1	s_2	s_3	s_4	:	**b**
2	2	0	1	0	0	0	:	7
0	2	1	0	1	1	0	:	12
3	1	–1	0	0	1	0	:	15
–2	0	1	0	0	0	1	:	8
4	2	–1	0	0	0	0	:	0

(d) If we would normally solve an LP as a maximisation problem, how could we solve: minimise $z = 4x_1 + 2x_2 - x_3$?

(e) (i) What was the first constraint **before** the slack variable was included? [1]
(ii) Why is the slack variable needed? [2]

(f) Which of columns 1,2 or 3 is it possible to pivot on? Explain.

(g) Select a pivot column. Which of rows 1,2,3,4 is it possible to pivot on? Explain.

(h) **Using your selected pivot** in (g), show the basic columns, the row containing your pivot and the objective row in the following augmented matrix).

QUESTION 2 [5 + 2 + 3 + 3 + 4 + 3 = 20 marks]

This question involves phase I Simplex and/or the big M method.

(a) By introducing both slack and artificial variables as needed, convert these two constraints so that they can be used with Simplex, with explanations:

(i) $2x_1 - 3x_2 + x_3 \geq 4$
(ii) $x_1 - x_2 + 3x_3 = 8$

(b) What is the fundamental difference between a phase I and phase II problem?

(c) Here is a simplex phase 1 classic (ie not big M) augmented matrix:

x_1	x_2	x_3	s_1	a_1	a_2	:	b
1	2	0	1	2	0	:	8
0	2	1	0	4	1	:	12
0	0	1	0	1	0	:	8
0	–1	–1	0	–2	0	:	–3

What would your conclusion be to this problem, and why?

(d) What would your conclusion be if at the end of phase 1 (either classic or big M methods) your matrix was this?

x_1	x_2	x_3	s_1	:	b
1	–2	0	0	:	8
0	–1	0	1	:	12
0	–3	1	0	:	8
0	1	–1	0	:	–3

(e) In phase I (classic or big M) we do not use the stated objective. Instead how do we proceed? You might use this constraint as an illustration : $3x_1 + x_2 - x_3 + s_1 + a_1 = 5$

(f) What condition in classic phase I or big M allows us to proceed to phase II? (Note. The answer is different for each method, of course. One answer only required.)

QUESTION 3 [7 + 5 + 3 = 15 marks]

This question involves the LP:

$$x_1 + x_2 \leq 5$$
$$x_1 - x_2 = 2$$
$$\max z = 3x_2$$

(a) Draw a graph to represent and solve the LP:

(b) Write down the dual of the LP:

(c) Use your result in (a) to find the optimal value of the dual in (b), with an explanation.

QUESTION 4 [3 + 3 + 7 + 2 = 15 marks]

The solution of a phase II LP (say A) is:

$x_1 = 3.5 \quad x_2 = 0.6 \quad s_2 = 5.5 \quad z\ (\max) = 8.2$

The **dual** of LP A is:

$$p_1 + p_2 + 2p_3 \geq 5$$
$$p_1 - p_2 - 3p_3 \geq 2$$
$$\min z = 2p_1 + 3p_2 + p_3$$

(a) How many slack variables did the original LP have (explaining why)?

(b) Write down the complementary slackness theorem, with an illustration of what it means using the above example.

(c) Use the complementary slackness theorem to deduce the value of three of the real/slack variables in the dual, and use this to solve for p_1, p_2 and p_3 in the dual LP.

(d) Show that your solution in (c) gives the expected minimum value of z.

QUESTION 5 [12 + 8 = 20 marks

(a) You have a choice. Select **one** of the following five algorithmic problems:
the **allocation problem** (with 5 items and 5 recipients),
the **travelling salesman problem** (with 5 cities),
the **transportation problem** (with 5 producers and 5 consumers),
the **minimum network cost problem** (with 6 nodes), or
the **maximum network flow problem** (with 6 nodes).

For your selected problem:

(i) Set up the problem (ie, introduce an example to work on).
(ii) Deduce a non-optimal initial feasible solution.
(iii) Show two steps of the appropriate hand algorithm to find the optimal solution.

(b) Write down the full LP (constraints and objective) corresponding to your model in (a).

--------- End of Examination ---------