THE PAPUA NEW GUINEA
UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
SECOND SEMESTER EXAMINATIONS 2022

FOURTH YEAR BACHELOR OF SCIENCE IN COMPUTER SCIENCE

CS421 - OPERATIONS RESEARCH

TIME ALLOWED – 3 HOURS

INFORMATION FOR CANDIDATES:

1. Write your name, student number, and program of study clearly on the front page of your answer booklet. Do it **now**.
2. You have 10 minutes to read this examination paper. During this time you must **NOT** write **inside** your answer booklet. You can make notes on the examination paper.
3. A scientific calculator is permitted, though **you do not have to use one.** Other electronic devices are not permitted. Notes and headphones are not permitted.
4. At the conclusion of the examination you must **immediately** put your pens down. You are **NOT** permitted to write inside your answer booklet after the "end of examination" announcement.
5. You can answer the questions in any order. Start each question on a new page. After you have finished the exam, indicate the order in which you answered questions in the left column of the marks box on the cover of the answer booklet.
6. There are 5 questions. Total marks is 100.

MARKING SCHEME:

Here is a marks matrix

Question	1	2	3	4	5
Total marks	20	15	15	10	40

The marks breakdown for question parts are indicated at the top of each question. The marks for question sub-parts are in brackets at the end of each sub-part question.

Question 1 [10 +10 = 20 marks]

Below is a linear program (LP) that involves 2 variables x and y and 3 constraints:

$$\begin{aligned} \max \quad & z = 2x - y \\ \text{where} \quad & 3x + 2y \le 30 \\ & -x + y \le 4 \\ & x - y \le 4 \qquad x, y \ge 0 \end{aligned}$$

(a) Solve the LP using the 2D graphical method. [Two of the corner points are (4,9) and (8.3).]

(b) If the classic Simplex method was used to solve this LP the phase II algorithm could be employed.

(i) What feature of the LP that allows phase II to be used. [2]

(ii) What is the theoretical reason why solving using the phase II algorithm is MUCH simpler than solving using phase I? [2]

(iii) Make a change to this LP so that it could NOT be solved using the phase II algorithm, stating what makes it not suitable for phase II. [2]

(iv) What would the phase I algorithm need to find in your new problem in (iii)? [No calculations required.][2]

(v) Redraw your diagram in (a). In this diagram illustrate (without doing any calculations) one way in which simplex phase II might find the optimal solution. [2]

QUESTION 2 [2 + 2 + 2 + 2 + 5 + 2 = 15 marks]

The linear program in question 1 was solved using the Simplex phase II algorithm.

(a) A simplex formulation introduces slack variables. Give one example of a slack variable using the problem's original constraints.

After the first step the solution matrix was (using our class notation)

0	5	1	0	−3	|	15
0	0	0	1	1	|	10
1	−1	0	0	1	|	5
0	1	0	0	−2	|	−10

(b) In this matrix, which columns (1,2,3,4,5) correspond to the real and the slack variables?

(c) In the same matrix, which columns (1,2,3,4,5) correspond to the basic and the non-basic variables?

(d) Each step of the Simplex algorithm requires a pivot. Explain how the pivot for the **next step** of the algorithm (from the above matrix) is selected.

(e) Perform the next algorithm step on the above matrix.

(f) Using the new matrix in (e) only, has the optimal solution been found yet? Explain.

QUESTION 3 [2 + 4 + 2 + 2 + 2 + 3 = 15 marks]

Below is a small primal linear program:

$$\begin{aligned} \max\ z = &\ x + 4y \\ \text{where}\quad &\ x - 3y \le 4 \\ &\ 2x + y \le 4 \qquad\qquad x,y \ge 0 \end{aligned}$$

After introducing two slack variables and solving using the Simplex algorithm, the solution was found to be: $x = 0$, $y = 4$, $s1=16$, $s2 = 0$ and $z = 16$.

(a) What do you understand by the dual of a mathematical problem?

(b) Write down the dual formulation of the LP above, using variables p and q.

(c) What would be the z-value of the optimal dual LP in (b)?
[No working, but an explanation needed]

(d) Convert the LP in (c) to a form in which the inequalities are replaced by equalities.
This formulation should use variables p, q, t1 and t2.

(e) In terms of a primal and dual linear program pair, what do understand by the "complementary slackness theorem"?

(f) Use the complementary slackness theorem, and a little algebra, to deduce the values of p, q, t1 and t2 in the optimum solution of the dual.

QUESTION 4 [2 + 3 + 2 + 3 = 10 marks]

(a) What do you understand by a "Monte Carlo" simulation?

(b) Illustrate your answer in (a) using this problem: A new digital network accessing the Internet needs to be installed. At any point in time there will be x users, and each user will require y kilobytes of bandwidth. Both x and y will change randomly every second. How much bandwidth should the network be capable of handling?

(c) A simulation computer program used the Python function shown on the right:
In terms of the simulation,
what is the purpose of this function?

```
def stream() :
   x = random.random()
   if x < 0.1 : s = 4
   elif  x < 0.3 : s = 6
   elif  x < 0.5 : s = 8
   elif  x < 0.7 : s = 7
   elif  x < 0.9 : s = 5
   else : s = 1
   return s
```

(d) What might be displayed if the function in (c) was used with the main program on the right? Explain.

```
i = 0
while i < 4 :
   print( stream() )
   i = i + 1
```

QUESTION 5 [8 + 8 + 8 + 8 + 8 = 40 marks]

This question involves the various graph/network algorithms looked at in class.

(a) A network needs to transport units from two sources A,B to three destinations P,Q,R.
A and B produce 20 and 10 units respectively.
P, Q and R require 10, 15 and 5 units respectively.
The unit cost matrix (shipping from source to destination) is:

	P	Q	R
A :	5	6	4
B :	2	1	3

(i) A matrix of initial "flows" suitable for our algorithm is given on the right:
How were these flows obtained? [2]

	P	Q	R
A :	10	10	-
B :	-	5	5

(ii) What is the total transportation cost using this initial set of flows? [2]

(iii) By switching the flow around a suitable loop, find a better solution (ie, with lower transportation cost). Show your working, and write down the cost (which if you proceeded correctly will be optimum). [4]

(b) In this problem each item (A,B,C) on the left needs to be paired with an item (P,Q,R) along the top .
The cost of each pairing is shown in the table.

	P	Q	R
A :	5	6	4
B :	10	12	8
C :	3	3	6

Find the optimal (lowest cost) allocation using the hand algorithm used in class (don't simply write down the answer!).

(c) Here is the cost matrix for a "travelling salesman" type problem:

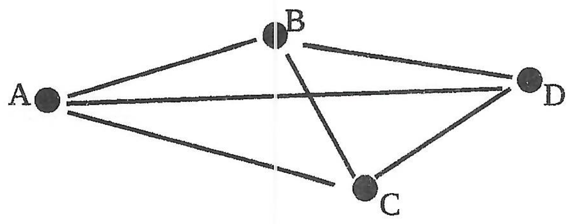
	A	B	C	D	E	F
A :	-	12	16	17	10	7
B :	12	-	14	5	3	9
C :	16	14	-	6	15	11
D :	17	5	6	-	8	4
E :	10	3	15	8	-	13
F :	7	9	11	4	13	-

Every node must be visited once, ending at the starting node. The total cost of the links traversed is to be a minimum.

(i) We looked at two algorithms. The simpler one is sometimes called the greedy algorithm. The other is called the insertion algorithm.
Show three steps of the **greedy algorithm** to illustrate how it works. [7]

(ii) True or false: The solution found using the greedy algorithm will be optimal (ie, have the lowest overall cost)? [1]

(d) In the network sketched below we require the maximum possible flow from A to D, where the symmetric maximum link flows are shown in the table.



Costs are		
	A↔B	9
	A↔C	5
	A↔D	6
	B↔C	3
	B↔D	4
	C↔D	4

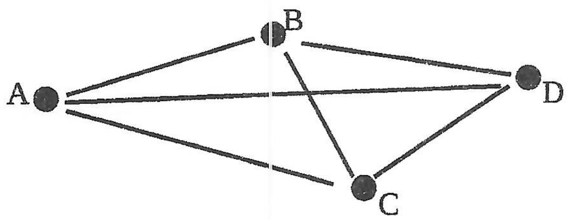
(i) You should be able to find four "cuts" separating node D from node A. Copy the above diagram, and illustrate these cuts. [2]

(ii) Write down the flow across each cut. [2]

(iii) What is the "max-flow, min-cut" theorem, and how does it, with your calculations in (ii) help find the maximum flow through the network? [3]

(iv) What extra useful information can the "max-flow, min-cut" theorem **not** find? [1]

(e) We require the minimum cost from node A to node D in the network sketched below. The symmetric maximum individual link costs are shown in the table.



Link costs are		
	A↔B	9
	A↔C	5
	A↔D	13
	B↔C	3
	B↔D	4
	C↔D	10

Show, step by step, how Dijkstra's algorithm can be used to solve this problem.

END OF EXAMINATION