PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATION - OCTOBER 2023

FIRST YEAR BACHELOR OF SCIENCE IN COMPUTER SCIENCE

CS122 - MATHEMATICS FOR COMPUTER SCIENCE

TIME ALLOWED: 3 HOURS

• INFORMATION FOR CANDIDATES:

1. Write your name and student number clearly on the front of the examination answer booklets.

2. You have 10 minutes to read this paper. You must not begin writing during this time.

3. This paper contains **eight (8)** questions. You must attempt all. The questions can be done in any order but all parts to the same question must be kept together.

4. Show all working.

5. All answers must be written in examination answer booklet(s) provided. No other written materials will be accepted.

6. Do not use red ink.

7. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.

8. Scientific and business calculators are allowed in the examination room.

9. The last pages contains information sheet for student use.

• MARKING SCHEME

Marks are indicated in brackets for each question. Total is **100 marks** with 50% weight.

Question 1 (8+4=12 marks)

a) Let $\vec{a} = \langle 3, 2, -1 \rangle$, $\vec{b} = \langle 0, -1, 3 \rangle$. Find a vector $\vec{c}$ orthogonal to both $\vec{a}$ and $\vec{b}$.

b) Graph vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ on a single set of axes.

Question 2 (10+6=16 marks)

a) Use *Gaussian elimination or Gauss-Jordan* to solve the following system of equations.

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ \qquad x_2 + x_3 = -1 \\ x_1 - x_2 - x_3 = -1 \end{cases}$$

b) Rotate the unit vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ by 90° about *z*-axis.

Question 3 (3+4+3=10 marks)

a) Convert the following binary number to its *decimal* representation. $(11100.0011)_2$

b) Convert the following decimal number to its *octal* representation. $(87.0767)_{10}$ up to 3 decimal places

c) Convert the following hexadecimal number to its *binary* representation. $(15C.AF)_{16}$

Question 4 (10+8=18 marks)

a) Use a truth table to verify $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$.

b) Then prove it in Boolean algebra.

Question 5 (12 marks)

Evaluate X(2), X(3) and X(4) for the recursively defined sequence (Tower of Hanoi problem):

$$\begin{cases} X(1) = 1 \\ X(n) = 2\,X(n-1) + 1, \qquad n > 1 \end{cases}$$

Then use recursive definition to prove the non-recursive formula ($X(n) = 2^n - 1$) by *induction*.

Question 6 (4+4+4=12 marks)

A committee of 3 people is to be chosen from a group of 4 men and 3 women.

i. In how many ways can we arrange a committee of manager, chairman and speaker?

How many committees are possible if:

ii. There are to be 2 men and 1 woman?

iii. There are to be women only?

Question 7 (8 marks)

Prove by contradiction method.

Proposition. For all $x > 0$, $x + \frac{1}{x} \geq 2$.

Question 8 (6+6=12 marks)

a) Construct an expression tree for the expression: $a + ((b - c) \times d)$

b) By applying a pre-order traversal to the expression tree, write the expression.

Reference Material

1) Cross product Formula: $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3 , a_1b_2 - a_2b_1 \rangle$$

2)

Gaussian Elimination method:

Reduced-row echelon form of the Augmented matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$ must be

$$\begin{bmatrix} 1 & a_{12} & a_{13} & b_1 \\ 0 & 1 & a_{23} & b_2 \\ 0 & 0 & 1 & b_3 \end{bmatrix}.$$

Gauss-Jordan Method:

Reduced-row echelon form of the Augmented matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$ must be

$$\begin{bmatrix} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 1 & b_3 \end{bmatrix}.$$

Rotation matrix about z-axis $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4) *Aaxioms*

Distributive Axiom: $x \times (y + z) = (x \times y) + (x \times z),$

$x + (y \times z) = (x + y) + (x + z)$

Inverse Axiom $x \times x' = 0\ ,\ x + x' = 1$

Identity Axiom $x \times 1 = x\ , x + 0 = x$

5) A *proof by induction* consists of two steps:
1. In the *base step*, prove that $P(n)$ is true when $n = 1$, that is, prove $P\ (1)$.
2. In the *inductive step*, prove that if $P(n)$ is true for any particular value of n, then it is also true for the next value of n.

6)

Permutation formula $$_nP_r = \frac{n!}{(n-r)!}$$

Combination formula $$_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$0! = 1, \qquad 1! = 1$$

7) *Proof by contradiction* steps:

1. Assume your statement to be false.
2. Proceed as you would with a direct proof.
3. Come across a contradiction.
4. State that because of the contradiction, it can't be the case that the statement is false, so it must be true.

8)

Pre-order traversal steps:

Step 1: Visit the root vertex.
Step 2: Traverse the left sub tree, i.e, traverse recursively.
Step 3: Traverse the right sub tree, i.e, traverse recursively.
Step 4: Repeat Steps 2 and 3 until all the vertices are visited.