THE PNG UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL & COMMUNICATIONS ENGINEERING

FIRST (1st) SEMESTER EXAMINATION (2021)

EE211 ELECTROMAGNETIC FIELD THEORY

TIME ALLOWED: 3 HOURS

INFORMATION FOR STUDENTS:

1. You have **TEN (10) MINUTES** to read this paper. Do not write during this allocated time
2. There are **a total of six (6) Questions** in this Exam Booklet. **Answer ANY Two Questions from Part A and ANY Two Questions from Part B.**
3. All answers must be written in the **Answer Booklet**
4. **COMPLETE STUDENT DETAILS ARE TO BE FILLED ON THE ANSWER BOOKLET-DO THIS NOW**
5. Only drawing instruments and calculators are allowed on your desk. Textbooks and notebooks are **NOT** allowed
6. If you are found **Cheating** in this Exam, penalties specified by the **University** shall be applied.
7. TURN OFF all your mobile phones and place them on the floor under your seat before you start the examination

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PART A. ANSWER ANY TWO QUESTIONS

QUESTION ONE

a) Explain with diagrams the addition of three vectors using associative law of vectors

[5 marks]

b) Explain with diagrams and equations the Cross-Product of Two Vectors

[5 marks]

c) A vector field is specified as G = 24xyax + 12(x2 + 2)ay + 18z2az. Given two points, P (1, 2, −1) and Q (−2, 1, 3), find:

(i) G at P

(ii) a unit vector in the direction of G at Q:

(iii) a unit vector directed from Q toward P:

(iv) the equation of the surface on which |G| = 60

[15 marks]

[TOTAL 25 Marks]

QUESTION TWO

a) The region in which $4 < r < 5$, $0 < \theta < 25°$ and $0.9\pi < \phi < 1.1\pi$ contains the volume charge density of $p_v = 10(r-4)(r-5)\sin\theta\sin(\theta/2)$. Outside the region $p_v = 0$. Find the charge within the region.

[10 marks]

b) A uniform line charge of 16nC/m is located along the line defined by $y = -2$, $z = 5$. If $\varepsilon = \varepsilon_o$

(i) Find **E** at P(1,2,3)

(ii) Find **E** at that point in the $z = 0$ plane where the direction of **E** is given by

$$(1/3)a_y - (2/3)a_z$$

[15 marks]

[TOTAL 25 Marks]

QUESTION THREE

a) The cylindrical surfaces $\rho = 1,2,3$ cm carry uniform surface charge densities of $20,-8,5$ nC/m^2 respectively

(i) How much electric flux passes through the closed surface $\rho = 5$ cm, $0 < z < 1$ m?

(ii) Find **D** at $P(1cm,2cm,3cm)$

[10 marks]

b) Within the spherical shell, $3 < r < 4$ m, the electric flux density is given as $D = 5(r-3)^3 a_r$ C/m^2

(i) What is the volume charge density at r = 4?

(ii) What is the electric flux density at $r = 4$?

(iii) How much electric flux leaves the sphere at $r = 4$?

(iv) How much charge is contained within the sphere $r = 4$?

[15 mark]
[Total 25 marks]

PART B. ANSWER ANY TWO QUESTIONS

QUESTION FOUR

a) Obtain a relationship between the conductivity and permittivity of a material and the signal frequency, the frequency at which the conduction current is equal to the displacement current.

[5 Marks]

b) A wireless power transfer systems uses a coil to capture the electromagnetic power transmitted to the receiver. A 20 cm x 20 cm square coil has 10 turns and is placed on the xy-plane with its center at the origin (0, 0) of the xy-coordinates. It is cut by an electromagnetic field wave. The magnetic field of the wave is parallel to the z-axis. The coil is placed in free space. The magnetic field is given by $E = 300\cos(10^3 t)$ MV/m. If the coil is terminated at a resistance of 1000 ohm, find:

(i) the current in the coil and

(ii) the electric power dissipated in the resistor.

[10 Marks]

c) Consider an imaginary box a x b x c placed in the x-y-z coordinates.

(i) Determine the net power flux P(t) entering the box due to a plane wave in the air given by $E = \mathbf{u}_x E_o \cos(\omega t - ky)$ V/m.

(ii) Determine the net time average power entering the box. (Note: **u** stands for unit vector.)

[10 Marks]
[Total 25 Marks]

QUESTION FIVE

a) From the four Maxwell's equations obtain the energy relations inside an electromagnetic wave given by the Poynting Theorem. Explain each term of the equation and its importance in electric power and communication engineering systems or devices.

[10 Marks]

b) A 60 MHz plane wave travelling in the -x-direction (negative x-direction) in dry soil with a relative permittivity of 9 has an electric field polarized along the z-direction. Assuming the dry soil to be lossless, and given that the magnetic field has a peak value of 10 mA/m and that its value was measured to be 7 mA/m at t=0 and x = -0.75 m, develop complete expression for the wave's electric and magnetic fields

[15 Marks]
[Total 25 Marks]

QUESTION SIX

a) The electric field and free space intensity of an electromagnetic wave propagating in a lossless medium with relative permittivity 9 and free space permeability is given by

$$\mathbf{E}(z,t) = -\ \mathbf{u}_y\ 37.7\cos(108t - kz + \pi/4)\ \text{(V/m)}$$

where **u** stands for unit vector.

Find:

(i) magnetic field intensity H(z,t) and

(ii) *k*.

[12 Marks]

b) At 2 GHz the conductivity of meat is 2 S/m. When a material is placed inside a microwave oven and the field is activated, the presence of electromagnetic waves in the conducting meat causes energy dissipation in the material in the form of meat.

(i) Develop an expression for the time average power per mm^2 dissipated in a material of conductivity σ if the peak electric field in the material is Eo.

(ii) If the power to be dissipated in the meat should be 1.6 W/mm^3, determine the electric field required.

[13 Marks]
[Total 25 Marks]

Electromagnetic Formula Sheet

Vector-Analysis:

Elements	Rectangular	Cylindrical	Spherical
dl	$dxa_x + dya_y + dza_z$	$dra_r + rd\emptyset a_\emptyset + dza_z$	$dRa_R + R\,d\theta a_\theta + R\sin\theta d\emptyset a_\emptyset$
dS	$dydza_x + dzdxa_y + dxdya_z$	$rd_\emptyset dza_r + drdza_\emptyset + rdrd_\emptyset a_z$	$R^2\sin\theta d\emptyset d\theta a_R + R\sin\theta d\emptyset dR a_\theta + RdRd\theta a_\emptyset$
dV	$dxdydz$	$rdrd_\emptyset dz$	$R^2\sin\theta d\emptyset d\theta dR$

Cylindrical to Rectangular and vice versa

$$\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix}=\begin{bmatrix}\cos\emptyset & -\sin\emptyset & 0\\ \sin\emptyset & \cos\emptyset & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}Ar\\A\emptyset\\Az\end{bmatrix},\quad \begin{bmatrix}Ar\\A\emptyset\\Az\end{bmatrix}=\begin{bmatrix}\cos\emptyset & \sin\emptyset & 0\\ -\sin\emptyset & \cos\emptyset & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix},$$

$x = r\cos\emptyset, y = r\sin\emptyset, r = x^2 + y^2,$
$\emptyset = \tan\frac{y}{x}$

Spherical to Rectangular and vice versa

$$\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix}=\begin{bmatrix}\sin\theta\cos\emptyset & \cos\theta\cos\emptyset & -\sin\emptyset\\ \sin\theta\sin\emptyset & \cos\theta\sin\emptyset & \cos\emptyset\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}AR\\A\theta\\A\emptyset\end{bmatrix},\quad \begin{bmatrix}AR\\A\theta\\A\emptyset\end{bmatrix}=\begin{bmatrix}\sin\theta\cos\emptyset & \sin\theta\sin\emptyset & \cos\theta\\ \cos\theta\cos\emptyset & \cos\theta\sin\emptyset & -\sin\theta\\ -\sin\emptyset & \cos\emptyset & 0\end{bmatrix}\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix},$$

$x = R\sin\theta\cos\emptyset, y = R\sin\theta\cos\emptyset,$
$z = R\cos\theta, R = x^2 + y^2 + z^2,$
$\theta = \tan^{-1}(R/\sqrt{(x^2+y^2)})$

Coordinate Systems	Gradient	divergence
Rectangular	$\nabla V = \frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z$	$\nabla . A = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$
Cylindrical	$\nabla V = \frac{\partial V}{\partial r}a_r + \frac{\partial V}{r\partial\emptyset}a_\emptyset + \frac{\partial V}{\partial z}a_z$	$\nabla . A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial}{r\partial\emptyset}A_\emptyset + \frac{\partial}{\partial z}A_z$
Spherical	$\nabla V = \frac{\partial V}{\partial R}a_R + \frac{\partial V}{R\partial\theta}a_\theta + \frac{\partial V}{R\sin\theta\partial\emptyset}a_\emptyset$	$\nabla V = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2A_R) + \frac{1}{R\sin\theta}\frac{\partial}{R\partial\theta}(A_\theta\sin\theta) + \frac{\partial}{R\sin\theta\partial\emptyset}A_\emptyset$

Electrostatics:

$F = \frac{Q_1Q_2}{4\pi\epsilon_0 R^2}a_{R12}$, $F = qE$, $\nabla\cdot E = \frac{\rho_v}{\epsilon_0}$, $\nabla\times E = 0$, $\oint_S E.ds = \frac{Q}{\epsilon_0}(gauss's\ law)$, $\oint_C E.dl = 0$, $E = \frac{Q}{4\pi\epsilon_0 R^2}a_R$, $p = qd$,

$E = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{R^2}dv\,a_R$, $E = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho R}{R^3}dv$, $E = \frac{1}{4\pi\epsilon_0}\int_S \frac{\rho_S}{R^2}ds\,a_R$, $E = \frac{1}{4\pi\epsilon_0}\int_L \frac{\rho_l}{R^2}dl\,a_R$, $E = \frac{\rho_l}{2\pi\epsilon_0 r}a_r$, $E = \frac{\rho_s}{2\epsilon_0}a_n$, $E = -\nabla V$,

$V = \frac{Q}{4\pi\epsilon_0 R}$, $V = \frac{W}{Q} = -\int_A^B E.dl$, $D = \epsilon_0 E$, $Q = \oint_S D.ds = \int \rho_v dv$, $\nabla\cdot D = \rho_v$, $V_{dipole} = \frac{Qd\cos\theta}{4\pi\epsilon R^2}$, $Q = CV$, $C = \frac{\epsilon s}{d}$,

$C = \frac{2\pi\epsilon L}{\ln(\frac{b}{a})}(for\ cylindrical\ capacitor)$, $\nabla^2 V = -\frac{\rho}{\epsilon}(poisson's\ equation)(\nabla^2\ is\ laplacian\ operator)$,

$\nabla^2 V = 0(Laplace\ equation)(no\ charge\ is\ present\ \rho = 0)$

Magnetostatics:

$I = \frac{\Delta q}{\Delta t}$, $I = \int J\cdot ds$, $J = \sigma E(A/m^2)(OHM's\ law\ in\ wave\ form)$, $\nabla . J = -\frac{\partial\rho}{\partial t}(Equation\ of\ continuity)(A/m^3)$,

$\nabla . J = 0(for\ steady\ current\ divergenceless)$, $\oint_S J\cdot ds = 0$, $\nabla\times\left(\frac{J}{\sigma}\right) = 0$, $\oint_C \frac{1}{\sigma}J\cdot dl = 0$, $F_m = q(\mathbf{V}\times\mathbf{B})$,

$F = F_m + F_E = q(\mathbf{E} + \mathbf{V}\times\mathbf{B})(lorentz's\ force\ equation)$, $\nabla . B = 0$, $\nabla\times B = \mu_0 J$, $\oint_S B.ds = 0$(law of conservation of magnetic flux), $\oint_C B\cdot dl = \mu_0 I$, $B = \nabla\times A$, $\nabla\cdot A = 0$, $\nabla^2 A = -\mu_0 J$, $A = \frac{\mu_0 I}{4\pi}\oint_C \frac{dl'}{R}$, $B = \frac{\mu_0 I}{4\pi}\oint_C \nabla\times(\frac{dl'}{R})$, $B = \frac{\mu_0 I}{4\pi}\oint_C(\frac{dl'\times a_R}{R^2})$,

$B = \frac{\mu_0 I}{4\pi}\oint_C(\frac{dl'\times R}{R^3})$, $B = \mu H$, $M = \chi_m H$, $1 + \chi_m = \mu_r$, $\oint_C H\cdot dl = I\ (Ampere's\ circuital\ law)$, $\Phi = \oint_C A\cdot dl = \oint_S B\cdot ds$,

$B_\emptyset = \frac{\mu_0 I}{2\pi r}a_\emptyset(line\ of\ current)$, $a_\emptyset = a_l\times a_r$, $\Phi_{12} = L_{12}I_1$, $\Lambda_{12} = N_2\Phi_{12} = L_{12}I_1$, $F_m = I\int dl\times B$

Time Varying Fields and Maxwell's Equations:

$V_{emf} = -\frac{d\Phi}{dt}$, $V_{emf} = \oint E\cdot dl = -\int\frac{\partial B}{\partial t}\cdot ds$, $V_{emf} = -\oint(u\times B)\cdot dl$.

Maxwell equations

Differential form	Integral form	Significance
$\nabla\times E = -\frac{\partial B}{\partial t}$	$\oint E\cdot dl = -\frac{d\Phi}{dt}$	Faraday's Law
$\nabla\times H = J + \frac{\partial D}{\partial t}$	$\oint H\cdot dl = I + \oint_S \frac{\partial D}{\partial t}\cdot ds$	Ampere's circuital Law
$\nabla\cdot D = \rho_v$	$\oint D\cdot ds = Q$	Gauss's Law
$\nabla . B = 0$	$\oint B\cdot ds = 0$	No isolated magnetic charge

Electromagnetic Formula Sheet

Vector-Analysis:

Elements	Rectangular	Cylindrical	Spherical
dl	$dx a_x + dy a_y + dz a_z$	$dr a_r + r d\phi a_\phi + dz a_z$	$dR a_R + R d\theta a_\theta + R\sin\theta d\phi a_\phi$
dS	$dydz a_x + dzdx a_y + dxdy a_z$	$r d_\phi dz a_r + dr dz a_\phi + r dr d_\phi a_z$	$R^2 \sin\theta d\phi d\theta a_R + R\sin\theta d\phi dR a_\theta + R dR d\theta a_\phi$
dV	$dxdydz$	$r dr d_\phi dz$	$R^2 \sin\theta d\phi d\theta dR$

Cylindrical to Rectangular and vice versa

$$\begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ar \\ A\phi \\ Az \end{bmatrix}, \quad \begin{bmatrix} Ar \\ A\phi \\ Az \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix},$$

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Coordinate Systems	Gradient	divergence
Rectangular	$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$	$\nabla . A = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$
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Spherical	$\nabla V = \frac{\partial V}{\partial R} a_R + \frac{\partial V}{R\partial\theta} a_\theta + \frac{\partial V}{R\sin\theta\partial\phi} a_\phi$	$\nabla V = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{R\partial\theta}(A_\theta \sin\theta) + \frac{\partial}{R\sin\theta\partial\phi} A_\phi$

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$\nabla^2 V = 0$(Laplace equation)(no charge is present $\rho = 0$)

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$I = \frac{\Delta Q}{\Delta t}$, $I = \int J \cdot ds$, $J = \sigma E (A/m^2)$(OHM'slaw in waveform), $\nabla . J = -\frac{\partial\rho}{\partial t}$ (Equation of continuity)(A/m^3),

$\nabla . J = 0$(for steady current divergenceless), $\oint_S J \cdot ds = 0$, $\nabla \times \left(\frac{J}{\sigma}\right) = 0$, $\oint_C \frac{1}{\sigma} J \cdot dl = 0$, $F_m = q(\mathbf{V} \times \mathbf{B})$,

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Cylindrical to Rectangular and vice versa

$$\begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ar \\ A\phi \\ Az \end{bmatrix}, \quad \begin{bmatrix} Ar \\ A\phi \\ Az \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix}, \quad x = r\cos\phi, y = r\sin\phi, r = x^2 + y^2, \ \phi = \tan\frac{y}{x}$$

Spherical to Rectangular and vice versa

$$\begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} AR \\ A\theta \\ A\phi \end{bmatrix}, \quad \begin{bmatrix} AR \\ A\theta \\ A\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix},$$

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Coordinate Systems	Gradient	divergence
Rectangular	$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$	$\nabla . A = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$
Cylindrical	$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{\partial V}{r\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z$	$\nabla . A = \frac{1}{r}\frac{\partial}{\partial r}(r A_r) + \frac{\partial}{r\partial \phi} A_\phi + \frac{\partial}{\partial z} A_z$
Spherical	$\nabla V = \frac{\partial V}{\partial R} a_R + \frac{\partial V}{R\partial \theta} a_\theta + \frac{\partial V}{R\sin\theta\partial\phi} a_\phi$	$\nabla V = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{R\partial\theta}(A_\theta \sin\theta) + \frac{\partial}{R\sin\theta\partial\phi} A_\phi$

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$E = \frac{1}{4\pi\epsilon_o}\int_V \frac{\rho}{R^2} dv\, a_R$, $E = \frac{1}{4\pi\epsilon_o}\int_V \frac{\rho R}{R^3} dv$, $E = \frac{1}{4\pi\epsilon_o}\int_S \frac{\rho_s}{R^2} ds\, a_R$, $E = \frac{1}{4\pi\epsilon_o}\int_L \frac{\rho_l}{R^2} dl\, a_R$, $E = \frac{\rho_l}{2\pi\epsilon_o r} a_r$, $E = \frac{\rho_s}{2\epsilon_o} a_n$, $E = -\nabla V$,
$V = \frac{q}{4\pi\epsilon_o R}$, $V = \frac{W}{Q} = -\int_A^B E.dl$, $D = \epsilon_o E$, $Q = \oint_S D.ds = \int \rho_v dv$, $\nabla \cdot D = \rho_v$, $V_{dipole} = \frac{Qd\cos\theta}{4\pi\epsilon R^2}$, $Q = CV$, $C = \frac{\epsilon s}{d}$,
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Magnetostatics:

$I = \frac{\Delta Q}{\Delta t}$, $I = \int J \cdot ds$, $J = \sigma E (A/m^2)$(OHM'slaw in waveform), $\nabla . J = -\frac{\partial\rho}{\partial t}$ (Equation of continuity)(A/m^3),
$\nabla . J = 0$(for steady current divergenceless), $\oint_S J \cdot ds = 0$, $\nabla \times \left(\frac{J}{\sigma}\right) = 0$, $\oint_C \frac{1}{\sigma} J \cdot dl = 0$, $F_m = q(\mathbf{V} \times \mathbf{B})$,
$\mathrm{F} = F_m + F_E = q(\mathbf{E} + \mathbf{V} \times \mathbf{B})$(lorentz's force equation), $\nabla . B = 0$, $\nabla \times B = \mu_o J$, $\oint_S B.ds = 0$(law of conservation of magnetic flux), $\oint_C B \cdot dl = \mu_o I$, $B = \nabla \times A$, $\nabla \cdot A = 0$, $\nabla^2 A = -\mu_o J$, $A = \frac{\mu_o I}{4\pi}\oint_C \frac{dl'}{R}$, $B = \frac{\mu_o I}{4\pi}\oint_C \nabla \times (\frac{dl'}{R})$, $B = \frac{\mu_o I}{4\pi}\oint_C (\frac{dl' \times a_R}{R^2})$,
$B = \frac{\mu_o I}{4\pi}\oint_C (\frac{dl' \times R}{R^3})$, $B = \mu H$, $M = \chi_m H$, $1 + \chi_m = \mu_r$, $\oint_C H \cdot dl = I$ (Ampere'scircuital law), $\Phi = \oint_C A \cdot dl = \oint_S B \cdot ds$,
$B_\phi = \frac{\mu_o I}{2\pi r} a_\phi$(line of current), $a_\phi = a_l \times a_r$, $\Phi_{12} = L_{12} I_1$, $\Lambda_{12} = N_2 \Phi_{12} = L_{12} I_1$, $F_m = I \int dl \times B$

Time Varying Fields and Maxwell's Equations:

$V_{emf} = -\frac{d\Phi}{dt}$, $V_{emf} = \oint E \cdot dl = -\int \frac{\partial b}{\partial t} \cdot ds$, $V_{emf} = -\oint (u \times B) \cdot dl$,

Maxwell equations

Differential form	Integral form	Significance
$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E \cdot dl = -\frac{d\Phi}{dt}$	Faraday's Law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint H \cdot dl = I + \oint_s \frac{\partial D}{\partial t} \cdot ds$	Ampere's circuital Law
$\nabla \cdot D = \rho_v$	$\oint D \cdot ds = Q$	Gauss's Law
$\nabla . B = 0$	$\oint B \cdot ds = 0$	No isolated magnetic charge