THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND COMMUNICATIONS ENGINEERING

FIRST SEMESTER EXAMINATION (2022)

EE411 CONTROL SYSTEMS

ELECTRICAL ENGINEERING – YEAR 4

APPLIED PHYSICS – YEAR 4

TIME ALLOWED: 3 HOURS

INFORMATION FOR STUDENTS:

- You have **TEN (10)** minutes to read the paper. You must **NOT** begin writing during this time.
- All answers must be written in the **ANSWER BOOK** supplied. **COMPLETE THE DETAILS REQUIRED ON THE FRONT COVER OF YOUR ANSWER BOOK. DO THIS NOW.**
- Drawing instruments and calculators are permitted.
- Answer **ALL FIVE (5)** questions.
- All questions carry equal marks (breakdown of the marks are indicated).
- If you are found cheating in the examination, the penalties specified by the University shall apply.
- Switch OFF all mobile phones.

Question One **Root Locus Method**

The open-loop transfer function, G(s), for a unity closed-loop control system is;

$$G(s)H(s) = \frac{K}{S(S+5)(S+10)}$$

Estimate and sketch the root locus for the system as K varies from 0 to infinity. Show ALL important calculations and features such as asymptotes, breakaway points, angles of departure, instability frequencies, etc. *(10)*

Question Two **Nyquist Diagram Method**

The stability of a closed-loop control system is to be investigated through the frequency response of the Nyquist method. The open-loop transfer function of the system is given below and the resulting frequency response is shown in Table 2.0.

$$G(s)H(s) = \frac{50}{S(S+2)(S+10)}$$

Table 2.0

ω (rad/s)	(j ω)	(1+ j0.5ω)	(1+ j0.1ω)	Gain (dB)	Phase (degree)
2.0	$2.0\angle 90^0$	$1.41\angle 45^0$			
2.5	$2.5\angle 90^0$	$1.62\angle 51^0$			
3.0	$3.0\angle 90^0$	$1.80\angle 56^0$			
3.5	$3.5\angle 90^0$	$2.01\angle 60^0$			
4.0	$4.0\angle 90^0$	$2.23\angle 63^0$			
4.5	$4.5\angle 90^0$	$2.51\angle 66^0$			
5.0	$5.0\angle 90^0$	$2.69\angle 68^0$			
5.5	$5.5\angle 90^0$	$2.97\angle 70^0$			
7.0	$7.0\angle 90^0$	$3.64\angle 74^0$			
8.5	$8.5\angle 90^0$	$4.42\angle 77^0$			
10	$10\angle 90^0$	$5.10\angle 79^0$			
20	$20\angle 90^0$	$10.05\angle 84^0$			

Complete the above table to determine the overall gains and phase angles and;

(i) plot the Nyquist diagram on the polar graph paper, and *(5)*
(ii) determine the gain and phase crossover frequencies and *(2)*
(iii) determine the gain margin and the phase margin. *(3)*

<u>Question Three</u>   <u>Bode Plot Method</u>

The open-loop frequency response of a control system is represented by the attached Bode plots of **Fig. 3.1**(see page 5) and **Fig. 3.2** (see page 6). From the Bode plots determine;

(i) the gain and phase crossover frequencies; *(4)*

(ii) the gain margin and the phase margin; *(6)*

<u>Question Four</u>   <u>Phase Lead and Phase Lag Compensator</u>

Consider the following closed-loop system in Fig. 4.1 and it's bode plot in Fig.4.2 for a system transfer function, *G(s)* given below;

$$G(s) = \frac{10}{s(s+10)}$$

Design the controller $G_c(s)$ so that the gain crossover frequency ω_g is between 40-60 rads/sec and phase margin (PM) is greater than 60°. *(10)*

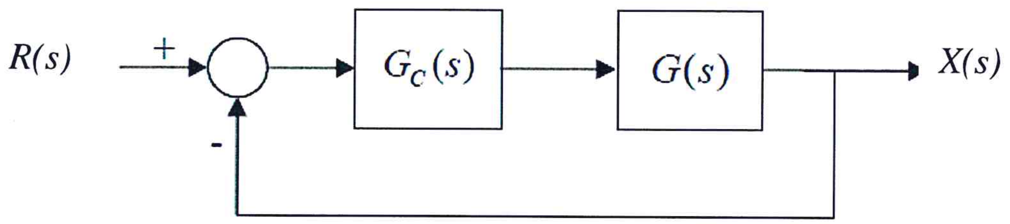


Fig. 4.1

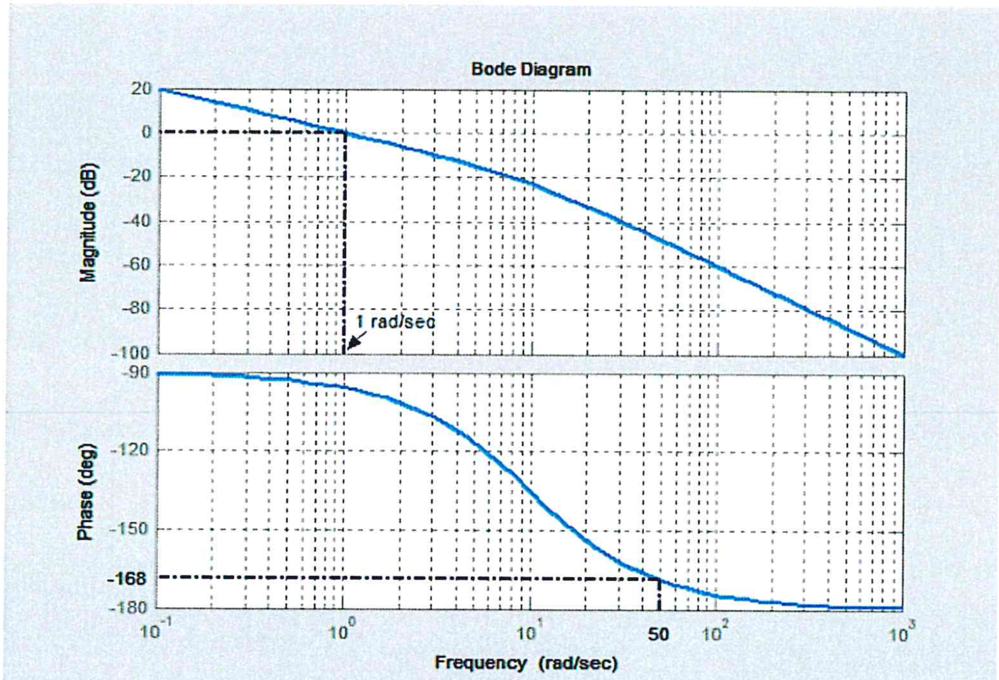


Fig. 4.2

Question Five   Digital Control Systems - Z Transform

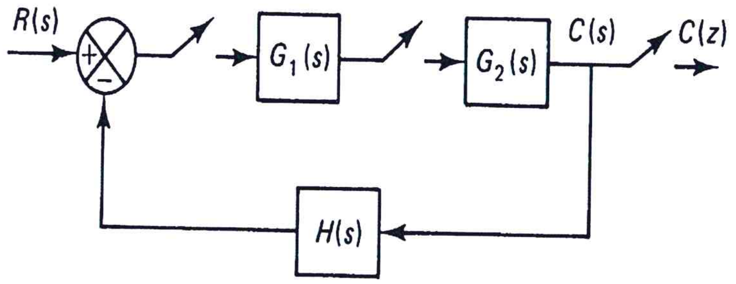
(a) A discrete time system is describe by the following difference equation with initial condition;

$$y(k+2) - 0.4y(k+1) + 0.05y(k) = (0.6)^{k+1}$$

with the initial conditions $y(0) = 0,\ y(1) = 1.$

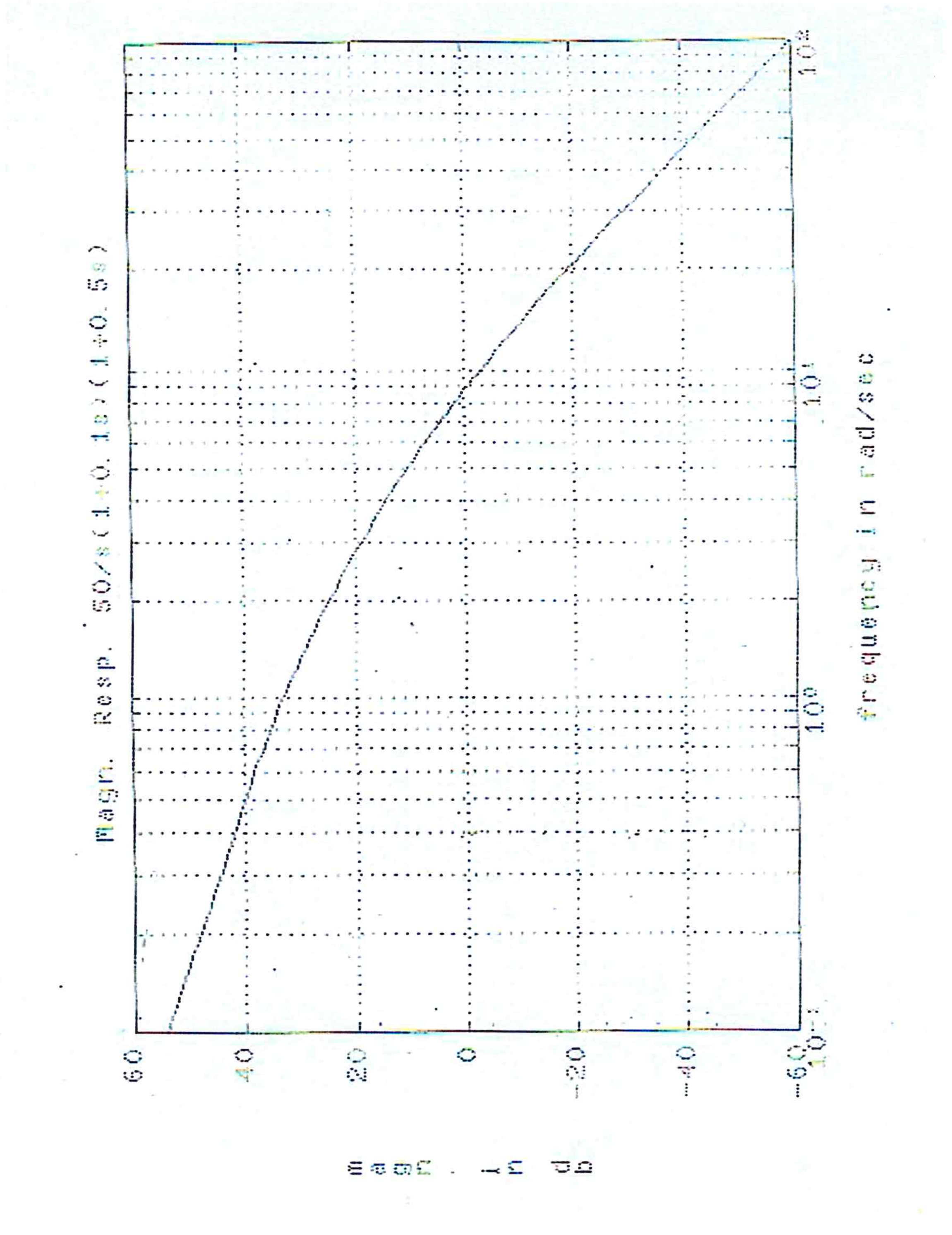
Find the solution $y(k)$ for $k > 0$ **(5)**

(b) For the closed-loop control system with the samplers in Fig. 5.0, derive its transfer function $C(z)/R(s)$. Show step by step derivation of its output shown below. *(5)*



$$C(z) = \frac{G_1(z)G_2(z)R(z)}{1 + G_1(z)G_2H(z)}$$

Fig. 5.0



Magn. Resp. 50/s(1+0.1s)(1+0.5s)
Frequency in rad/sec
Magn. in dB
60
40
20
0
-20
-40
-60
10⁰
10¹
10²

Fig. 3.1

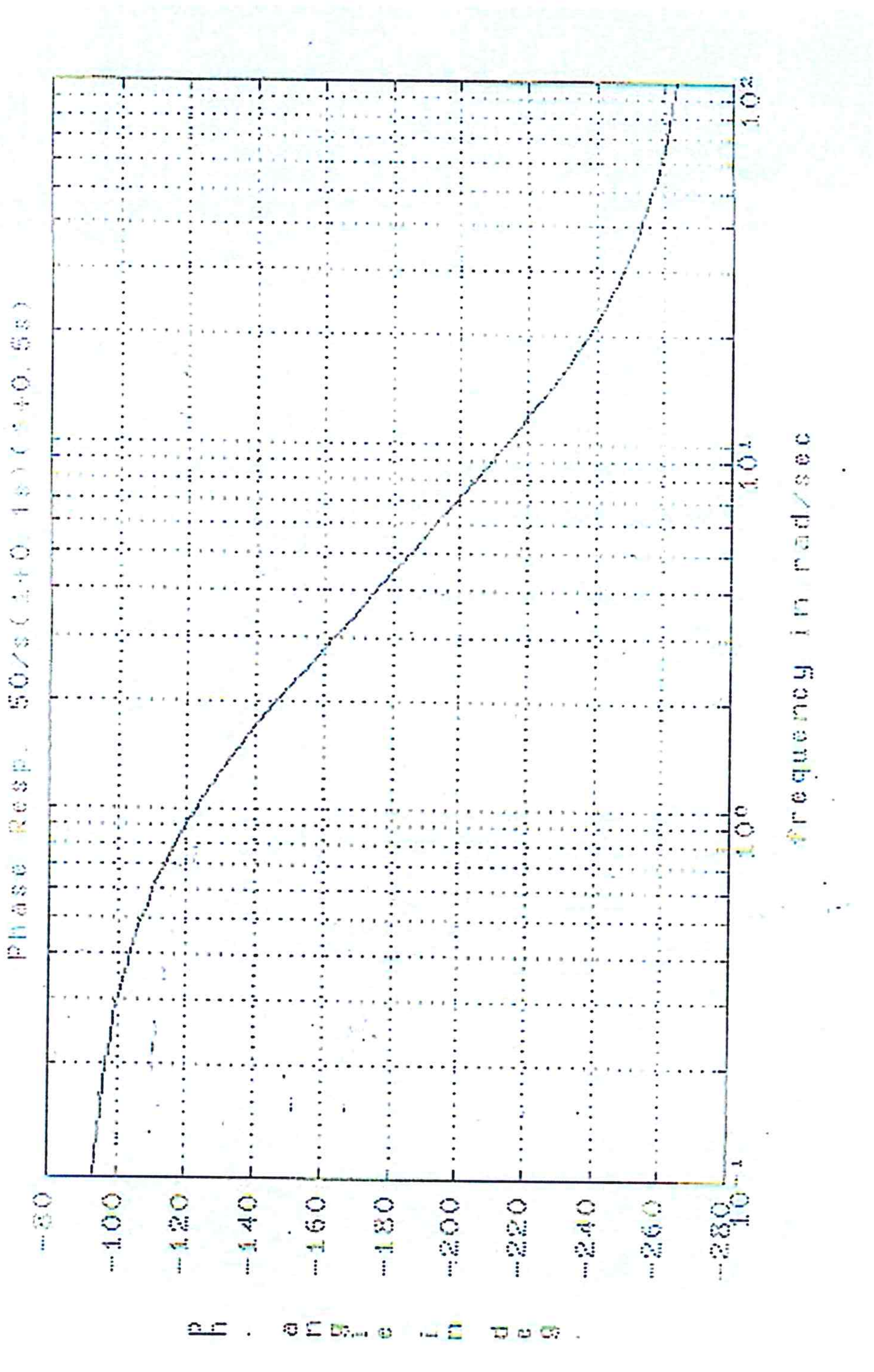
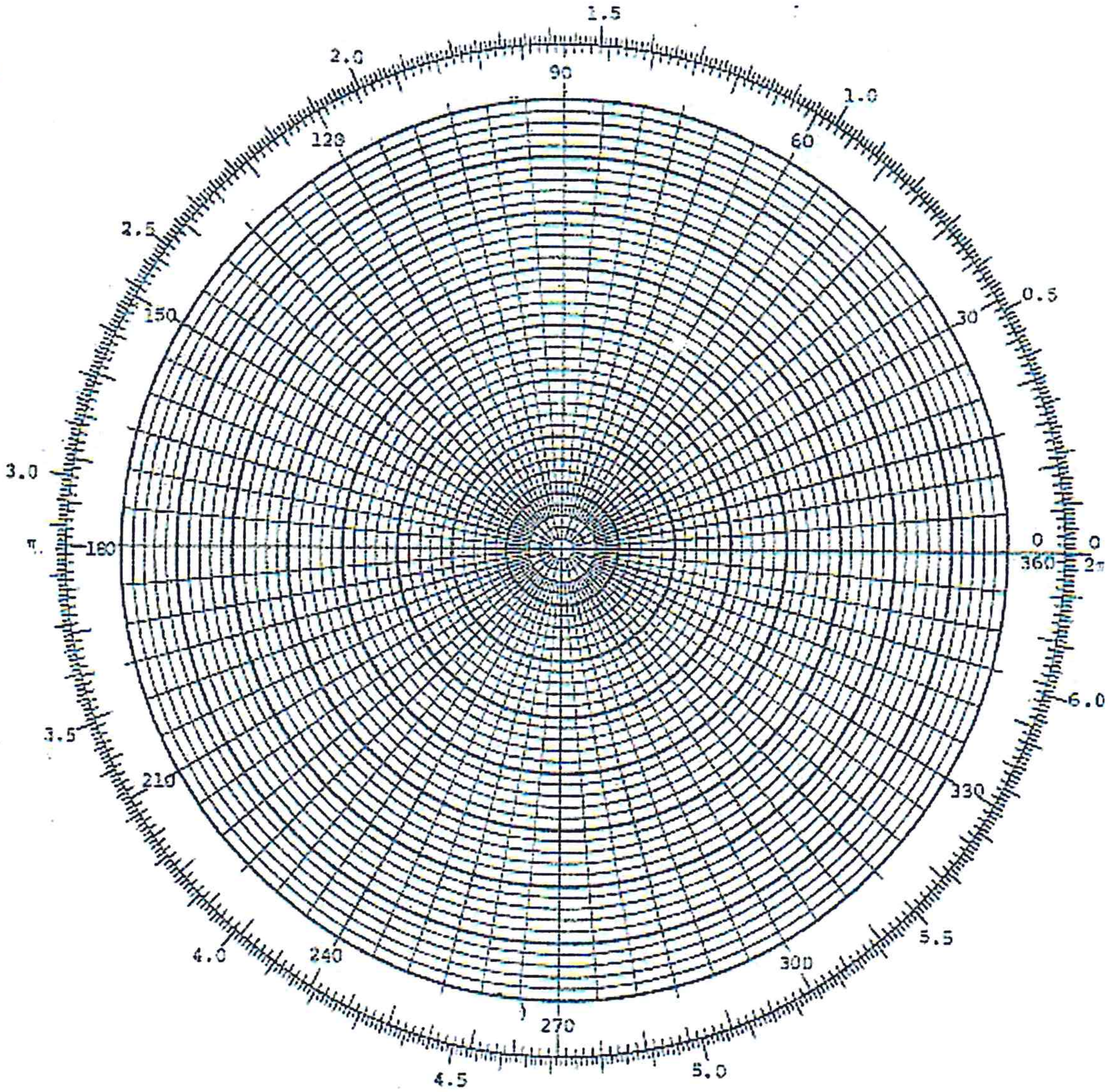
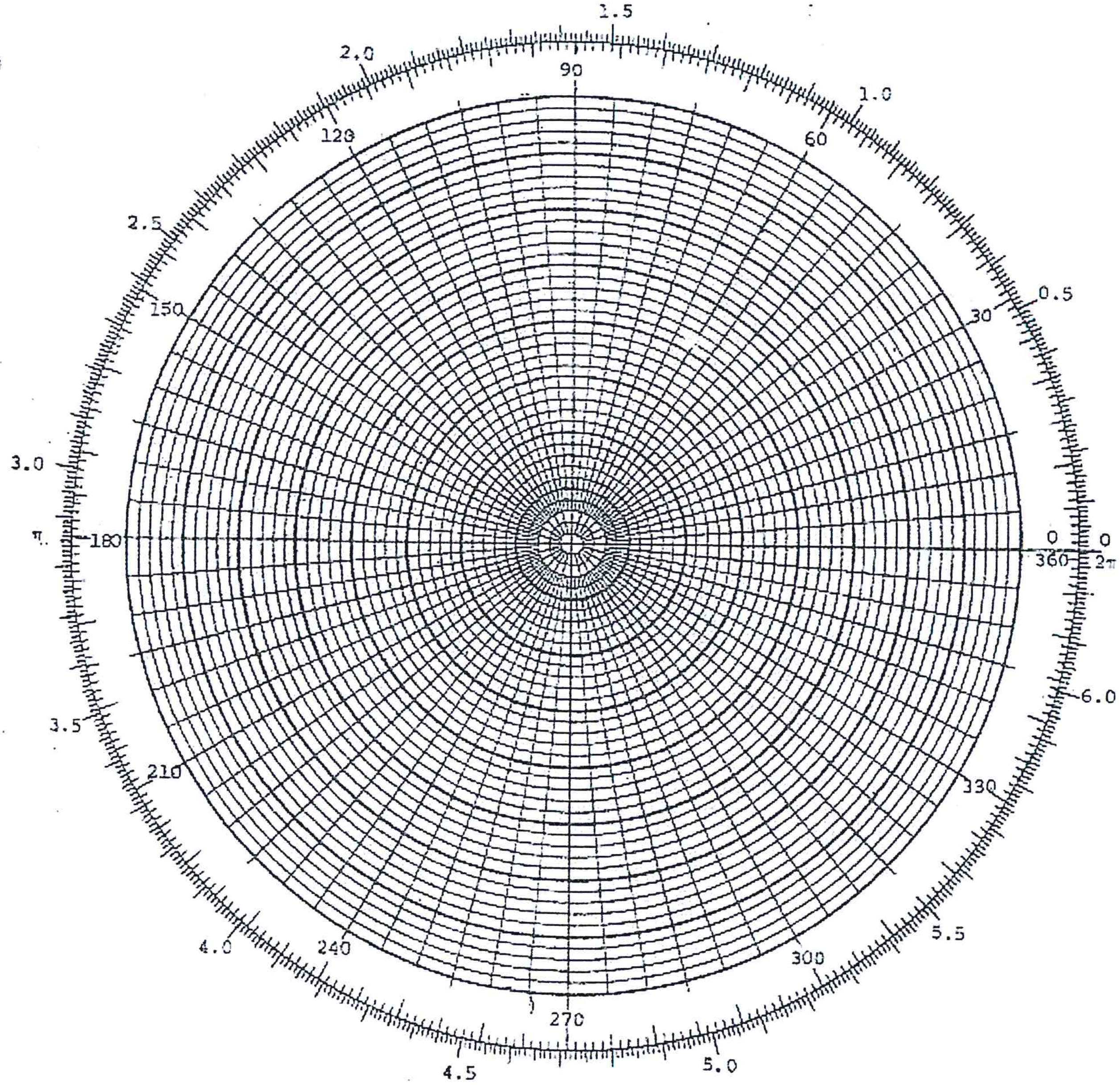


Fig. 3.2


1.5
90
2.0
1.0
120
60
2.5
0.5
150
30
3.0
π
180
0
360
0
2π
6.0
3.5
210
330
4.0
240
300
5.5
270
4.5
5.0


1.5
2.0
90
1.0
120
60
2.5
0.5
150
30
3.0
π
180
0
0
360
2π
6.0
3.5
210
330
5.5
4.0
240
300
270
4.5
5.0

Appendix 1
Complex Number Table

For the complex number $z = 1 + jx$, the modulus M and the argument θ are such that

$$1 + jx = Me^{j\theta} = M\angle\theta^{\circ}, \text{ where } M = \sqrt{(1 + x^2)} \text{ and}$$

$$\theta = \tan^{-1}x$$

x	M	θ°	x	M	θ°	x	M	θ°
0.05	1.00	3	1.80	2.06	61	5.2	5.30	79
0.10	1.00	6	1.85	2.12	62	5.4	5.49	79
0.15	1.01	8	1.90	2.15	62	5.6	5.69	80
0.20	1.02	11	1.95	2.18	63	5.8	5.89	80
0.25	1.03	14	2.00	2.23	63	6.0	6.08	81
0.30	1.04	17	2.1	2.32	64	6.2	6.28	81
0.35	1.06	19	2.2	2.42	66	6.4	6.48	81
0.40	1.08	22	2.3	2.51	66	6.6	6.68	81
0.45	1.09	24	2.4	2.60	67	6.8	6.87	82
0.50	1.12	27	2.5	2.69	68	7.0	7.07	82
0.55	1.14	29	2.6	2.79	69	7.2	7.27	82
0.60	1.17	31	2.7	2.88	70	7.4	7.47	82
0.65	1.19	33	2.8	2.97	70	7.6	7.67	82
0.70	1.22	35	2.9	3.07	71	7.8	7.87	83
0.75	1.25	37	3.0	3.16	72	8.0	8.06	83
0.80	1.28	39	3.1	3.26	72	8.2	8.26	83
0.85	1.31	40	3.2	3.35	73	8.4	8.46	83
0.90	1.34	42	3.3	3.45	73	8.6	8.66	83
0.95	1.38	43	3.4	3.54	74	8.8	8.86	83
1.00	1.41	45	3.5	3.64	74	9.0	9.05	84
1.05	1.45	46	3.6	3.74	75	9.2	9.25	84
1.10	1.49	48	3.7	3.83	75	9.4	9.45	84
1.15	1.52	49	3.8	3.94	75	9.6	9.65	84
1.20	1.55	50	3.9	4.02	76	10	10.05	84
1.25	1.62	51	4.0	4.12	76	12	12.0	85
1.30	1.64	52	4.1	4.22	76	15	15.0	86
1.35	1.67	53	4.2	4.32	77	20	20.0	87
1.40	1.72	54	4.3	4.42	77	30	30	88
1.45	1.76	55	4.4	4.51	77	40	40	89
1.50	1.80	56	4.5	4.61	77	50	50	89
1.55	1.88	57	4.6	4.71	78	60	60	89
1.60	1.89	58	4.7	4.81	78	70	70	89
1.65	1.93	59	4.8	4.90	78	80	80	89
1.70	1.97	59	4.9	5.00	78	90	90	89
1.75	2.01	60	5.0	5.10	79	100	100	89
						∞	∞	90

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1 - at)e^{-at}$	$(1 - akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$

Definition of the Z-transform

$$\mathscr{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$Z\{x(t)\}$ or $Z\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t)+bx_2(t)$	$aX_1(z)+bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z)-zx(0)$
4.	$x(t+2T)$	$z^2X(z)-z^2x(0)-zx(T)$
5.	$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
6.	$x(t+kT)$	$z^kX(z)-z^kx(0)-z^{k-1}x(T)-\ldots-zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z)-z^kx(0)-z^{k-1}x(1)-\ldots-zx(k1-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz\frac{d}{dz}X(z)$
11.	$kx(k)$	$-z\frac{d}{dz}X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^{a})$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z\frac{d}{dz}X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z\to\infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z\to 1}\left[\left(1-z^{-1}\right)X(z)\right]$ if $\left(1-z^{-1}\right)X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k)=x(k)-x(k-1)$	$\left(1-z^{-1}\right)X(z)$
19.	$\Delta x(k)=x(k+1)-x(k)$	$(z-1)X(z)-zx(0)$
20.	$\sum_{k=0}^{n}x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a}x(t,a)$	$\frac{\partial}{\partial a}X(z,a)$
22.	$k^mx(k)$	$\left(-z\frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^{n}x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty}x(k)$	$X(1)$