THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL & COMMUNICATION ENGINEERING

BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING YEAR 4 –COMMUNICTIONS (BEEC 4) AND POWER (BEEP 4) AND BACHELOR OF SCIENCE IN APPLIED PHYSICS (BASP 4)

EE425: DIGITAL CONTROL SYSTEMS

FIRST SEMESTER EXAMINATION – 2021

TIME ALLOWED: 3 HOURS

INFORMATION FOR STUDENTS

1. You have **TEN (10)** minutes to read the paper.
   You must **NOT** begin writing during this time.

2. All answers must be written in the **ANSWER BOOK** supplied. **COMPLETE THE DETAILS REQUIRED ON THE FRONT COVER OF YOUR ANSWER BOOK. DO THIS NOW!**

3. Drawing instruments and calculators are allowed.

4. Attempt **ALL** questions.

5. The total number of marks for the paper is **100.** All questions carry equal marks

6. If anyone is found cheating in the Examinations, the penalties specified by the University shall apply.

QUESTION ONE (20)

A certain 2nd order linear time invariant lumped system is represented by the following generalized z-transform.

$$Y(z)=\frac{\left[-2y(-1)+y(-1)z^{-1}+y(-2)\right]}{3+2z^{-1}-z^{-2}}+\frac{\left(2z^{-1}-3z^{-2}\right)}{3+2z^{-1}-z^{-2}}U(z)$$
$$=\underbrace{\frac{\left[-2y(-1)z^{2}+y(-1)z+y(-2)z^{2}\right]}{3z^{2}+2z-1}}_{\text{Zero-Input Response}}+\underbrace{\frac{(2z-3)}{3z^{2}+2z-1}}_{\text{Zero-State Response}}U(z)$$

Where *y(-1)* and *y(-2)* two initial conditions and *U(z)* is the z-transform of the input signal *u(k)*. Supposing this system has the following initial conditions, ***y(-1) = 1*** and ***y(-2) = -2***, and is subjected to an input signal *u(k)* such that;

$$u(k)=\begin{cases}0 & \text{for } k<0\\ 1 & \text{for } k\geq 0\end{cases}$$

Solve the z-transform *(Y(z))* to find its time response for the given input signal

QUESTION TWO (20)

A certain digital control system can be represented by the generalized block diagram in Figure 1.

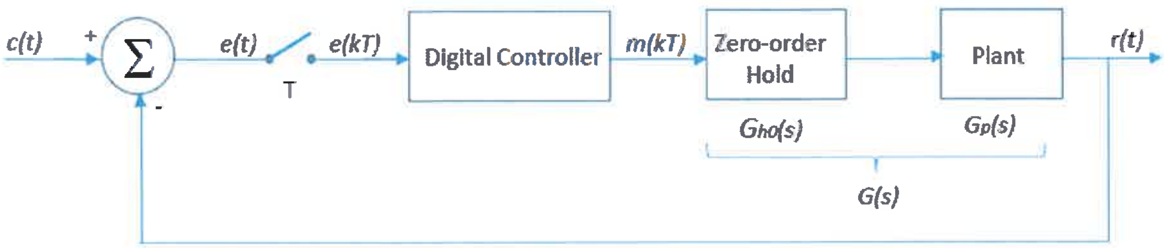

Figure 1: Block diagram of a Digital Control System (Q2)

For this system;

$$G_{h0}(s)=\frac{1-e^{-Ts}}{s}\text{, and } G_p(s)=\frac{1}{s^2+12s+20}$$

Find the z-transform *(G(z))* of *G(s)* as shown in Figure 1, assume $T = 1s$

QUESTION THREE (20)

Assuming the Digital Controller in Figure 1 (See Q2) is a Digital PID Controller. Obtain the positional form of the pulse transfer function for this Digital PID Controller *($G_D(z)$)* for the following gains, $K_P = 1$, $K_I = 0.2$ and $K_D = 0.2$,

Given

$$G_D(z) = K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)$$

QUESTION FOUR (20)

Obtain the closed-loop transfer function, $C(z)/R(z)$ for the system in Figure 1, assuming the Digital Control is the Digital PID Controller with the pulse transfer function determine in Q3.

QUESTION FIVE (20)

(a) Using the direct-division (long division) method obtain the inverse z-transform for the for a system defined by the z-transform given below;

$$F(z) = \frac{2z-3}{3z^2+2z-1}$$

Obtain the value of $f(k)$ for $k = 0,1,2$

(b) Using the Impulse Response Function Method, determine the z-transform for the continuous time transfer function of the system given below;

$$X(s) = \frac{2}{s(s+1)}$$

Take $T = 1s$

ALL DATA REQUIRED TO ASSIST YOU WITH THE EXAM IS INCLUDED

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{\left(1-e^{-aT}\right)z^{-1}}{\left(1-z^{-1}\right)\left(1-e^{-aT}z^{-1}\right)}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{\left(e^{-aT}-e^{-bT}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)\left(1-e^{-bT}z^{-1}\right)}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}\left(1+e^{-aT}z^{-1}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{\left[\left(aT-1+e^{-aT}\right)+\left(1-e^{-aT}-aTe^{-aT}\right)z^{-1}\right]z^{-1}}{\left(1-z^{-1}\right)^2\left(1-e^{-aT}z^{-1}\right)}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{\left(1-az^{-1}\right)^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}\left(1+az^{-1}\right)}{\left(1-az^{-1}\right)^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}\left(1+4az^{-1}+a^2z^{-2}\right)}{\left(1-az^{-1}\right)^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}\left(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3}\right)}{\left(1-az^{-1}\right)^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$

Definition of the Z-transform

$$\mathcal{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$Z\{x(t)\}$ or $Z\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t)+bx_2(t)$	$aX_1(z)+bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z)-zx(0)$
4.	$x(t+2T)$	$z^2X(z)-z^2x(0)-zx(T)$
5.	$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
6.	$x(t+kT)$	$z^kX(z)-z^kx(0)-z^{k-1}x(T)-\ldots-zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z)-z^kx(0)-z^{k-1}x(1)-\ldots-zx(k1-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz\frac{d}{dz}X(z)$
11.	$kx(k)$	$-z\frac{d}{dz}X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^{a})$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z\frac{d}{dz}X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z\to\infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$ if $(1-z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k)=x(k)-x(k-1)$	$(1-z^{-1})X(z)$
19.	$\Delta x(k)=x(k+1)-x(k)$	$(z-1)X(z)-zx(0)$
20.	$\sum_{k=0}^{n} x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a}x(t,a)$	$\frac{\partial}{\partial a}X(z,a)$
22.	$k^m x(k)$	$\left(-z\frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^{n} x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$