THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS - 2022

FIRST YEAR CIVIL AND MECHANICAL ENGINEERING AND COMPUTER SCIENCE

EN112A – ENGINEERING MATHEMATICS I

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. **Answer any five (5) questions out of six (6) questions.**
4. All answers must be written in examination booklets only. No other written material will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes and textbooks are not allowed in the examination room. All mobile phones and electronic/recording devices must be switched off during the examination.
7. Scientific calculators are allowed in the examination room.
8. A formula sheet is attached.

MARKING SCHEME

Marks are indicated at the beginning of each question. All questions carry equal marks.

Question 1 FUNCTIONS AND LIMITS (10 marks)

a) Given the identity $(\log_b a)(\log_a b) = 1$. Solve $(\log_2 81)(\log_3 32)$ without using a calculator. Show your working out. (2 marks)

b) What is the amplitude of $f(x) = \frac{1}{2}\cos x$? (2 marks)

c) If $\sinh x_0 = 2$, what is $\cosh x_0$? (2 marks)

d) Find $\lim_{x\to 2} \frac{x^2-4}{x-2}$ (2 marks)

e) First rationalize the numerator, then find $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$ (2 marks)

Question 2 DERIVATIVES (10 marks)

a) Given that $x = 1 + \sin\theta$ and $y = \sin\theta - \frac{1}{2}\cos 2\theta$, show that $\frac{d^2y}{dx^2} = 2$ (3 marks)

b) If $x^2y + xy^2 - x^3 - y^3 + 16 = 0$ find $\frac{dy}{dx}$ in its simplest form. Hence find the gradient of the normal to the curve at the point $(1, 3)$ (3 marks)

c) Find relative extrema using both the first and second derivative test, of $f(x) = \frac{1}{2}x - \sin x,\ 0 < x < 2\pi.$ (4 marks)

Question 3 SEQUENCE AND SERIES (10 marks)

a) The third term of an AP is 34 and the seventeenth term is -8 find the sum of the first twenty terms - use logical steps to find the relevant equation to solve. (3 marks)

b) Given the expansion of $\cos x$ is $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!}$ and the expansion of $(1+x)^{-1}$ is $\sum_{k=0}^{\infty}(-1)^k x^k$, give the series of $[\cos x \times (1+x)^{-1}]$? Show the first five terms at least. Show all working out. (3 marks)

c) Find the Maclaurin series for $f(x) = e^{-x}$ in sigma notation. (4 marks)

Question 4 INTEGRATION (10 marks)

a) Solve $I = \int \left(3x^2 + 1 + \frac{1}{x^2+x-2}\right) dx$. Complete integration of the 3rd term in the integral by partial fractions and complete the whole problem. (3 marks)

b) solve: $\int \sec^7 x \tan^3 x\, dx$. (3 marks)

c) Find the arc length of the curve $y = \frac{x^2}{2}$ from $x = 0$ to $x = 1$. Arc length of a curve is given by the formula $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$. Solve using trigonometric substitution. Draw the triangle to indicate the relevant working out. DO NOT USE ANY OTHER METHOD. Helpful table is given. (4 marks)

Question 5 COMPLEX NUMBERS (10 marks)

a) Find and **plot all roots** of the following complex number $\sqrt[4]{-7 + 24i}$. (7 marks)

b) Express each root in its rectangular, polar and exponential form. (3 marks)

Question 6 **PROBABILITY AND STATISTICS (10 marks)**

a) Find k in the probability function $f(x) = k\binom{3}{x}$, $x = 0,1,2,3$, and sketch f and the distribution function (CDF) F. (4 marks)

b) A die is rolled 5 times, determine the binomial probability of obtaining three sixes. (2 marks)

c) Classical experiments by E. Rutherford and H. Geiger in 1910 showed that the number of alpha particles emitted per second in a radioactive process is a random variable X having a Poisson distribution. If X has mean 0.5 seconds, what is the probability of observing two or more particles during any given second? (2 marks)

d) Let X be normal with mean 105 and variance 25. Find $\mathrm{P}(110.5 < \mathrm{X} < 111.25)$. (2 marks)

DATA SHEET for EN112A EXAMS 2022 SEMESTER 1

Trigonometrical identities

(a) $\sin^2\theta + \cos^2\theta = 1;\ \sec^2\theta = 1 + \tan^2\theta;\ \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(b) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

(c) Let $A = B = \theta \ \therefore \quad \sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Hyperbolic identities

$\cosh x + \sinh x = e^x$	$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$
$\cosh x - \sinh x = e^{-x}$	$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$
$\cosh^2 x - \sinh^2 x = 1$	$\sinh(x-y) = \sinh x\cosh y - \cosh x\sinh y$
$1 - \tanh^2 x = \operatorname{sech}^2 x$	$\cosh(x-y) = \cosh x\cosh y - \sinh x\sinh y$
$\coth^2 x - 1 = \operatorname{csch}^2 x$	$\sinh 2x = 2\sinh x\cosh x$
$\cosh(-x) = \cosh x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\sinh(-x) = -\sinh x$	$\cosh 2x = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$

Derivatives and Integrals

1	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\therefore \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$	$\left\{\begin{matrix}\text{provided}\\ n \neq -1\end{matrix}\right\}$
2	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\therefore \int \frac{1}{x}\,dx = \ln x + C$	
3	$\frac{d}{dx}(e^x) = e^x$	$\therefore \int e^x\,dx = e^x + C$	
4	$\frac{d}{dx}(e^{kx}) = ke^{kx}$	$\therefore \int e^{kx}\,dx = \frac{e^{kx}}{k} + C$	
5	$\frac{d}{dx}(a^x) = a^x \ln a$	$\therefore \int a^x\,dx = \frac{a^x}{\ln a} + C$	
6	$\frac{d}{dx}(\cos x) = -\sin x$	$\therefore \int \sin x\,dx = -\cos x + C$	
7	$\frac{d}{dx}(\sin x) = \cos x$	$\therefore \int \cos x\,dx = \sin x + C$	
8	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\therefore \int \sec^2 x\,dx = \tan x + C$	

More derivatives

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x\tan x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x\cot x$$

Specific integrals

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

TRIGONOMETRIC SUBSTITUTIONS

EXPRESSION IN THE INTEGRAND	SUBSTITUTION	RESTRICTION ON θ	SIMPLIFICATION
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$-\pi/2 \le \theta \le \pi/2$	$a^2 - x^2 = a^2 - a^2\sin^2\theta = a^2\cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$-\pi/2 < \theta < \pi/2$	$a^2 + x^2 = a^2 + a^2\tan^2\theta = a^2\sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\begin{cases}0 \le \theta < \pi/2 & (\text{if } x \ge a)\\ \pi/2 < \theta \le \pi & (\text{if } x \le -a)\end{cases}$	$x^2 - a^2 = a^2\sec^2\theta - a^2 = a^2\tan^2\theta$

Reduction formula

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$$

$$\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

INTEGRATING PRODUCTS OF TANGENTS AND SECANTS

$\int \tan^m x \sec^n x\, dx$	PROCEDURE	RELEVANT IDENTITIES
n even	• Split off a factor of $\sec^2 x$. • Apply the relevant identity. • Make the substitution $u = \tan x$.	$\sec^2 x = \tan^2 x + 1$
m odd	• Split off a factor of $\sec x \tan x$. • Apply the relevant identity. • Make the substitution $u = \sec x$.	$\tan^2 x = \sec^2 x - 1$
$\begin{cases} m \text{ even} \\ n \text{ odd} \end{cases}$	• Use the relevant identities to reduce the integrand to powers of $\sec x$ alone. • Then use the reduction formula for powers of $\sec x$.	$\tan^2 x = \sec^2 x - 1$

Mechanics Rule: $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$

Arithmetic series: $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series: $S_n = \frac{a(1-r)}{1-r}$

Roots of a complex number: $\sqrt[n]{z} = \left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$

The binomial expansion: $(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r = \sum_{r=0}^{n} {}^nC_r a^{n-r}b^r = \sum_{r=0}^{n}\frac{n!}{r!(n-r)!}a^{n-r}b^r$

- CDF is the integral of PDF for continuous $f(x)$ or the cummulation (sum) of the probability function (PMF) for discrete $f(x)$.
- The mean of a random distribution is given by:

$$\mu = \sum_j x_j f(x_j)$$

for discrete cases

Or

$$\mu = \int_{-\infty}^{\infty} x f(x)dx$$

for continuous cases.

- The Expectation according to the k^{th} Moment is given by,

$$E([X-\mu]^k) = \sum_j (x_j - \mu)^k f(x_j)$$

for discrete cases

Or

$$\int_{-\infty}^{\infty} (x-\mu)^k f(x)dx$$

for continuous cases.

The Poisson distribution: $P(x=r) = \frac{e^{-\mu}\mu^r}{r!}$ **The normal distribution:** $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$

Table A7 Normal Distribution

Values of the distribution function $\Phi(z)$ [see (3), Sec. 24.8]. $\Phi(-z) = 1 - \Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
	0.		**0.**		**0.**		**0.**		**0.**		**0.**
0.01	5040	0.51	6950	1.01	8438	1.51	9345	2.01	9778	2.51	9940
0.02	5080	0.52	6985	1.02	8461	1.52	9357	2.02	9783	2.52	9941
0.03	5120	0.53	7019	1.03	8485	1.53	9370	2.03	9788	2.53	9943
0.04	5160	0.54	7054	1.04	8508	1.54	9382	2.04	9793	2.54	9945
0.05	5199	0.55	7088	1.05	8531	1.55	9394	2.05	9798	2.55	9946
0.06	5239	0.56	7123	1.06	8554	1.56	9406	2.06	9803	2.56	9948
0.07	5279	0.57	7157	1.07	8577	1.57	9418	2.07	9808	2.57	9949
0.08	5319	0.58	7190	1.08	8599	1.58	9429	2.08	9812	2.58	9951
0.09	5359	0.59	7224	1.09	8621	1.59	9441	2.09	9817	2.59	9952
0.10	5398	0.60	7257	1.10	8643	1.60	9452	2.10	9821	2.60	9953
0.11	5438	0.61	7291	1.11	8665	1.61	9463	2.11	9826	2.61	9955
0.12	5478	0.62	7324	1.12	8686	1.62	9474	2.12	9830	2.62	9956
0.13	5517	0.63	7357	1.13	8708	1.63	9484	2.13	9834	2.63	9957
0.14	5557	0.64	7389	1.14	8729	1.64	9495	2.14	9838	2.64	9959
0.15	5596	0.65	7422	1.15	8749	1.65	9505	2.15	9842	2.65	9960
0.16	5636	0.66	7454	1.16	8770	1.66	9515	2.16	9846	2.66	9961
0.17	5675	0.67	7486	1.17	8790	1.67	9525	2.17	9850	2.67	9962
0.18	5714	0.68	7517	1.18	8810	1.68	9535	2.18	9854	2.68	9963
0.19	5753	0.69	7549	1.19	8830	1.69	9545	2.19	9857	2.69	9964
0.20	5793	0.70	7580	1.20	8849	1.70	9554	2.20	9861	2.70	9965
0.21	5832	0.71	7611	1.21	8869	1.71	9564	2.21	9864	2.71	9966
0.22	5871	0.72	7642	1.22	8888	1.72	9573	2.22	9868	2.72	9967
0.23	5910	0.73	7673	1.23	8907	1.73	9582	2.23	9871	2.73	9968
0.24	5948	0.74	7704	1.24	8925	1.74	9591	2.24	9875	2.74	9969
0.25	5987	0.75	7734	1.25	8944	1.75	9599	2.25	9878	2.75	9970
0.26	6026	0.76	7764	1.26	8962	1.76	9608	2.26	9881	2.76	9971
0.27	6064	0.77	7794	1.27	8980	1.77	9616	2.27	9884	2.77	9972
0.28	6103	0.78	7823	1.28	8997	1.78	9625	2.28	9887	2.78	9973
0.29	6141	0.79	7852	1.29	9015	1.79	9633	2.29	9890	2.79	9974
0.30	6179	0.80	7881	1.30	9032	1.80	9641	2.30	9893	2.80	9974
0.31	6217	0.81	7910	1.31	9049	1.81	9649	2.31	9896	2.81	9975
0.32	6255	0.82	7939	1.32	9066	1.82	9656	2.32	9898	2.82	9976
0.33	6293	0.83	7967	1.33	9082	1.83	9664	2.33	9901	2.83	9977
0.34	6331	0.84	7995	1.34	9099	1.84	9671	2.34	9904	2.84	9977
0.35	6368	0.85	8023	1.35	9115	1.85	9678	2.35	9906	2.85	9978
0.36	6406	0.86	8051	1.36	9131	1.86	9686	2.36	9909	2.86	9979
0.37	6443	0.87	8078	1.37	9147	1.87	9693	2.37	9911	2.87	9979
0.38	6480	0.88	8106	1.38	9162	1.88	9699	2.38	9913	2.88	9980
0.39	6517	0.89	8133	1.39	9177	1.89	9706	2.39	9916	2.89	9981
0.40	6554	0.90	8159	1.40	9192	1.90	9713	2.40	9918	2.90	9981
0.41	6591	0.91	8186	1.41	9207	1.91	9719	2.41	9920	2.91	9982
0.42	6628	0.92	8212	1.42	9222	1.92	9726	2.42	9922	2.92	9982
0.43	6664	0.93	8238	1.43	9236	1.93	9732	2.43	9925	2.93	9983
0.44	6700	0.94	8264	1.44	9251	1.94	9738	2.44	9927	2.94	9984
0.45	6736	0.95	8289	1.45	9265	1.95	9744	2.45	9929	2.95	9984
0.46	6772	0.96	8315	1.46	9279	1.96	9750	2.46	9931	2.96	9985
0.47	6808	0.97	8340	1.47	9292	1.97	9756	2.47	9932	2.97	9985
0.48	6844	0.98	8365	1.48	9306	1.98	9761	2.48	9934	2.98	9986
0.49	6879	0.99	8389	1.49	9319	1.99	9767	2.49	9936	2.99	9986
0.50	6915	1.00	8413	1.50	9332	2.00	9772	2.50	9938	3.00	9987