THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS - 2023

FIRST YEAR BACHELOR OF CIVIL AND MECHANICAL ENGINEERING AND COMPUTER SCIENCE

EN112A – ENGINEERING MATHEMATICS I

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. **Answer any five (5) questions out of six (6) questions.**
4. All answers must be written in examination booklets only. No other written material will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes and textbooks are not allowed in the examination room. All mobile phones and electronic/recording devices must be switched off during the examination.
7. Scientific calculators are allowed in the examination room.
8. A formula sheet is attached.

MARKING SCHEME

Marks are indicated at the beginning of each question. All questions carry equal marks.

Question 1 **FUNCTIONS AND LIMITS (10 marks)**

a) Find a formula for $(f \circ g \circ h)$ for the following functions:

$f(x) = x^2 + 1,\ g(x) = \frac{1}{x},\ h(x) = x^3.$ **(2 marks)**

b) Solve for x without using a calculator. Use the natural logarithm anywhere that logarithms are needed: $xe^{-x} + 2e^{-x} = 0.$ **(2 marks)**

c) If $\tanh x_0 = \frac{4}{5}$ what is $\sinh x_0$, and $\cosh x_0$? **(2 marks)**

d) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$. Use L'Hopital's rule. **(2 marks)**

e) Find the limit: $\lim_{t\to 2} \frac{t^3+3t^2-12t+4}{t^3-4t}$. **(2 marks)**

Question 2 **DERIVATIVES (10 marks)**

a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of $x = a\cos^3\theta, y = a\sin^3\theta$ **(5 marks)**

b) If $2x^2 + y^2 - 6y - 9x = 0$ determine the equation of the normal to the curve at point (1,7). **(5 marks)**

Question 3 **SEQUENCE AND SERIES (10 marks)**

a) Consider the sequence $\{a_n\}_{n=1}^{+\infty}$ where $a_n = \frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{n}{n^2}$

(i) Find a_1, a_2, a_3 and a_4.

(ii) Use numerical evidence to make a conjecture about the limit of the sequence.

(iii) Confirm your conjecture by expressing a_n in closed form and calculating the limit. **(5 marks)**

b) Find the first four distinct Taylor polynomials about $x = x_0$, for: $\ln(x+1)$; $x_0 = 0$ **(5 marks)**

Question 4 **INTEGRATION (10 marks)**

a) In the following evaluate the integral. Use trigonometric substitution:

$\int \frac{\sqrt{x^2-9}}{x} dx$ **(3 marks)**

b) Solve the following by method of partial fractions,

$\int \frac{2x+4}{x^3-2x^2} dx$ **(3 marks)**

c) Determine the surface area of the solid obtained by rotating $y = \sqrt{9-x^2}$, $-2 \le x \le 2$ about the x-axis. Sketch it also. **(4 marks)**

Question 5 **COMPLEX NUMBERS (10 marks)**

a) Find and **plot all roots** of the following complex number $\sqrt[4]{-7+25\sqrt{5}i}$. **(7 marks)**

b) Express each root in its rectangular, polar and exponential form. **(3 marks)**

Question 6 PROBABILITY AND STATISTICS (10 marks)

a) Let X [millimeters] be the thickness of washers a machine turns out. Assume that X has the density $f(x) = kx$ if $0.9 < x < 1.1$ and 0 otherwise. Find k. What is the probability that a washer will have thickness between 0.95mm and 1.05mm? **(3 marks)**

b) A machine produces on average 1.5 per cent defectives. In a random sample of 80 items determine the probability of three being defective. Compare the Poisson probability with the Binomial and comment. **(3 marks)**

c) Let X be normal with mean 3.6 and variance 0.01. Find c such that:

(i) $P(X \leqq c) = 50\%$, **(2 marks)**

(ii) $P(X > c) = 10\%$, **(2 marks)**

DATA SHEET for EN112A EXAMS 2023 SEMESTER 1

Trigonometrical identities

(a) $\sin^2\theta + \cos^2\theta = 1$; $\sec^2\theta = 1 + \tan^2\theta$; $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(b) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

(c) Let $A = B = \theta$ $\therefore$ $\sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Hyperbolic identities

$\cosh x + \sinh x = e^x$	$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$
$\cosh x - \sinh x = e^{-x}$	$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$
$\cosh^2 x - \sinh^2 x = 1$	$\sinh(x-y) = \sinh x\cosh y - \cosh x\sinh y$
$1 - \tanh^2 x = \operatorname{sech}^2 x$	$\cosh(x-y) = \cosh x\cosh y - \sinh x\sinh y$
$\coth^2 x - 1 = \operatorname{csch}^2 x$	$\sinh 2x = 2\sinh x\cosh x$
$\cosh(-x) = \cosh x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\sinh(-x) = -\sinh x$	$\cosh 2x = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$

Derivatives and Integrals

1 $\frac{d}{dx}(x^n) = nx^{n-1}$ $\therefore$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $\left\{\begin{matrix}\text{provided}\\ n \neq -1\end{matrix}\right\}$

2 $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\therefore$ $\int \frac{1}{x}\,dx = \ln x + C$

3 $\frac{d}{dx}(e^x) = e^x$ $\therefore$ $\int e^x\,dx = e^x + C$

4 $\frac{d}{dx}(e^{kx}) = ke^{kx}$ $\therefore$ $\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$

5 $\frac{d}{dx}(a^x) = a^x\ln a$ $\therefore$ $\int a^x\,dx = \frac{a^x}{\ln a} + C$

6 $\frac{d}{dx}(\cos x) = -\sin x$ $\therefore$ $\int \sin x\,dx = -\cos x + C$

7 $\frac{d}{dx}(\sin x) = \cos x$ $\therefore$ $\int \cos x\,dx = \sin x + C$

8 $\frac{d}{dx}(\tan x) = \sec^2 x$ $\therefore$ $\int \sec^2 x\,dx = \tan x + C$

More derivatives

$\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\sec x] = \sec x\tan x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\csc x] = -\csc x\cot x$

Specific integrals

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

TRIGONOMETRIC SUBSTITUTIONS

EXPRESSION IN THE INTEGRAND	SUBSTITUTION	RESTRICTION ON θ	SIMPLIFICATION
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$-\pi/2 \le \theta \le \pi/2$	$a^2 - x^2 = a^2 - a^2\sin^2\theta = a^2\cos^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$-\pi/2 < \theta < \pi/2$	$a^2 + x^2 = a^2 + a^2\tan^2\theta = a^2\sec^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$\begin{cases}0 \le \theta < \pi/2 & (\text{if } x \ge a)\\ \pi/2 < \theta \le \pi & (\text{if } x \le -a)\end{cases}$	$x^2 - a^2 = a^2\sec^2\theta - a^2 = a^2\tan^2\theta$

Reduction formula

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$$

$$\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$

- Equation of a straight line: $y - y_1 = m(x - x_1)$
- The gradient of the normal is equivalent to the negative reciprocal of the gradient of a tangent.
- Radius of curvature, $R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$
- Position of centre of curvature (h, k):
  $h = x_1 - R\sin\theta,$
  $k = y_1 + R\cos\theta$

INTEGRATING PRODUCTS OF TANGENTS AND SECANTS

$\int \tan^m x \sec^n x\, dx$	PROCEDURE	RELEVANT IDENTITIES
n even	• Split off a factor of $\sec^2 x$. • Apply the relevant identity. • Make the substitution $u = \tan x$.	$\sec^2 x = \tan^2 x + 1$
m odd	• Split off a factor of $\sec x \tan x$. • Apply the relevant identity. • Make the substitution $u = \sec x$.	$\tan^2 x = \sec^2 x - 1$
$\begin{cases} m \text{ even} \\ n \text{ odd} \end{cases}$	• Use the relevant identities to reduce the integrand to powers of $\sec x$ alone. • Then use the reduction formula for powers of $\sec x$.	$\tan^2 x = \sec^2 x - 1$

- Volume by cylindrical shells: $V_y = \int_a^b 2\pi x f(x)dx$ or $V_x = \int_a^b 2\pi y f(y)dy$
- Length of a curve $L = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$
- Mechanics Rule: $x_{n+1} = \frac{1}{2}\left(x_n + \frac{c}{x_n}\right)$
- Arithmetic series: $S_n = \frac{n}{2}[2a + (n-1)d]$
- Geometric series: $S_n = \frac{a(1-r^n)}{1-r}$
- Maclaurin series: $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$
- Taylor series:
  - $f(x) = f(x_0) + f'(x_0)(x - x_0) + f''(x_0)\frac{(x-x_0)^2}{2!} + f'''(x_0)\frac{(x-x_0)^3}{3!} + \cdots + f^{(n)}(x_0)\frac{(x-x_0)^n}{n!}$
- Roots of a complex number: $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$ where; $k = 0, 1, 2, \cdots$

The binomial expansion: $(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r = \sum_{r=0}^{n} {}^nC_r a^{n-r}b^r = \sum_{r=0}^{n}\frac{n!}{r!(n-r)!}a^{n-r}b^r$

- CDF is the integral of PDF for continuous $f(x)$ or the cummulation (sum) of the probability function (PMF) for discrete $f(x)$.
- The mean of a random distribution is given by:

$$\mu = \sum_j x_j f(x_j)$$

for discrete cases

Or

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$

for continuous cases.

- The Expectation according to the k^{th} Moment is given by,

$$E([X-\mu]^k) = \sum_j (x_j - \mu)^k f(x_j)$$

for discrete cases

Or

$$\int_{-\infty}^{\infty} (x-\mu)^k f(x) dx$$

for continuous cases.

The Poisson distribution: $P(x = r) = \frac{e^{-\mu}\mu^r}{r!}$

The normal distribution: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$

Table A7 Normal Distribution

Values of the distribution function $\Phi(z)$ [see (3), Sec. 24.8]. $\Phi(-z) = 1 - \Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
	0.		0.		0.		0.		0.		0.
0.01	5040	0.51	6950	1.01	8438	1.51	9345	2.01	9778	2.51	9940
0.02	5080	0.52	6985	1.02	8461	1.52	9357	2.02	9783	2.52	9941
0.03	5120	0.53	7019	1.03	8485	1.53	9370	2.03	9788	2.53	9943
0.04	5160	0.54	7054	1.04	8508	1.54	9382	2.04	9793	2.54	9945
0.05	5199	0.55	7088	1.05	8531	1.55	9394	2.05	9798	2.55	9946
0.06	5239	0.56	7123	1.06	8554	1.56	9406	2.06	9803	2.56	9948
0.07	5279	0.57	7157	1.07	8577	1.57	9418	2.07	9808	2.57	9949
0.08	5319	0.58	7190	1.08	8599	1.58	9429	2.08	9812	2.58	9951
0.09	5359	0.59	7224	1.09	8621	1.59	9441	2.09	9817	2.59	9952
0.10	5398	0.60	7257	1.10	8643	1.60	9452	2.10	9821	2.60	9953
0.11	5438	0.61	7291	1.11	8665	1.61	9463	2.11	9826	2.61	9955
0.12	5478	0.62	7324	1.12	8686	1.62	9474	2.12	9830	2.62	9956
0.13	5517	0.63	7357	1.13	8708	1.63	9484	2.13	9834	2.63	9957
0.14	5557	0.64	7389	1.14	8729	1.64	9495	2.14	9838	2.64	9959
0.15	5596	0.65	7422	1.15	8749	1.65	9505	2.15	9842	2.65	9960
0.16	5636	0.66	7454	1.16	8770	1.66	9515	2.16	9846	2.66	9961
0.17	5675	0.67	7486	1.17	8790	1.67	9525	2.17	9850	2.67	9962
0.18	5714	0.68	7517	1.18	8810	1.68	9535	2.18	9854	2.68	9963
0.19	5753	0.69	7549	1.19	8830	1.69	9545	2.19	9857	2.69	9964
0.20	5793	0.70	7580	1.20	8849	1.70	9554	2.20	9861	2.70	9965
0.21	5832	0.71	7611	1.21	8869	1.71	9564	2.21	9864	2.71	9966
0.22	5871	0.72	7642	1.22	8888	1.72	9573	2.22	9868	2.72	9967
0.23	5910	0.73	7673	1.23	8907	1.73	9582	2.23	9871	2.73	9968
0.24	5948	0.74	7704	1.24	8925	1.74	9591	2.24	9875	2.74	9969
0.25	5987	0.75	7734	1.25	8944	1.75	9599	2.25	9878	2.75	9970
0.26	6026	0.76	7764	1.26	8962	1.76	9608	2.26	9881	2.76	9971
0.27	6064	0.77	7794	1.27	8980	1.77	9616	2.27	9884	2.77	9972
0.28	6103	0.78	7823	1.28	8997	1.78	9625	2.28	9887	2.78	9973
0.29	6141	0.79	7852	1.29	9015	1.79	9633	2.29	9890	2.79	9974
0.30	6179	0.80	7881	1.30	9032	1.80	9641	2.30	9893	2.80	9974
0.31	6217	0.81	7910	1.31	9049	1.81	9649	2.31	9896	2.81	9975
0.32	6255	0.82	7939	1.32	9066	1.82	9656	2.32	9898	2.82	9976
0.33	6293	0.83	7967	1.33	9082	1.83	9664	2.33	9901	2.83	9977
0.34	6331	0.84	7995	1.34	9099	1.84	9671	2.34	9904	2.84	9977
0.35	6368	0.85	8023	1.35	9115	1.85	9678	2.35	9906	2.85	9978
0.36	6406	0.86	8051	1.36	9131	1.86	9686	2.36	9909	2.86	9979
0.37	6443	0.87	8078	1.37	9147	1.87	9693	2.37	9911	2.87	9979
0.38	6480	0.88	8106	1.38	9162	1.88	9699	2.38	9913	2.88	9980
0.39	6517	0.89	8133	1.39	9177	1.89	9706	2.39	9916	2.89	9981
0.40	6554	0.90	8159	1.40	9192	1.90	9713	2.40	9918	2.90	9981
0.41	6591	0.91	8186	1.41	9207	1.91	9719	2.41	9920	2.91	9982
0.42	6628	0.92	8212	1.42	9222	1.92	9726	2.42	9922	2.92	9982
0.43	6664	0.93	8238	1.43	9236	1.93	9732	2.43	9925	2.93	9983
0.44	6700	0.94	8264	1.44	9251	1.94	9738	2.44	9927	2.94	9984
0.45	6736	0.95	8289	1.45	9265	1.95	9744	2.45	9929	2.95	9984
0.46	6772	0.96	8315	1.46	9279	1.96	9750	2.46	9931	2.96	9985
0.47	6808	0.97	8340	1.47	9292	1.97	9756	2.47	9932	2.97	9985
0.48	6844	0.98	8365	1.48	9306	1.98	9761	2.48	9934	2.98	9986
0.49	6879	0.99	8389	1.49	9319	1.99	9767	2.49	9936	2.99	9986
0.50	6915	1.00	8413	1.50	9332	2.00	9772	2.50	9938	3.00	9987