THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS 2022

FIRST YEAR ELECTRICAL ENGINEERING
FIRST YEAR MINING ENGINEERING
FIRST YEAR MINERAL PROCESSING

EN112 - ENGINEERING MATHEMATICS 1

TIME ALLOWED – 3 HOURS

INFORMATION FOR CANDIDATES:

1. Write your name, student number, and program of study clearly on the front page of your answer booklet. Do it **now**.
2. You have 10 minutes to read this examination paper. During this time you must **NOT** write **inside** your answer booklet. You can make notes on the examination paper.
3. A scientific calculator is permitted, though **you do not have to use one.** Other electronic devices are not permitted. Notes and headphones are not permitted.
4. At the conclusion of the examination you must **immediately** put your pens down. You are **NOT** permitted to write inside your answer booklet after the "end of examination" announcement.
5. You can answer the questions in any order. Start each question on a new page. After you have finished the exam, indicate the order in which you answered questions in the left column of the marks box on the cover of the answer booklet.
6. There are 6 questions. **You should attempt only 5.** If you attempt all 6 only the first 5 will be marked.

MARKING SCHEME: Each question is worth 20 marks. The marks breakdown for question parts is indicated at the top of each question. You should attempt only 5 questions.

QUESTION 1 [4 + 6 + 10 = 20 marks]

(a) Decompose the function $y = 4 + 3\cos^2(x)$ into two simpler functions.

(b) (i) If $f(x) = 5\sqrt{x+3}+2$ find the inverse function $f^{-1}(x)$.

(ii) Write down the domain and range of both of $f(x)$ and $f^{-1}(x)$.

(c) Tide heights depend on the relative positions of the sun and moon. At a certain time of year the tide height in Lae might be modelled by $h(t) = 0.5 \sin(\pi/12\ t)$ where 'h' is the height above or below the middle (median) tide height in metres, and t is the time in hours.

(i) What is the tide height above or below the median at times t=6, 12 and 18 hours?

(ii) Sketch the tide height from t=0 hours to t=24 hours (ie, one full day)

(iii) Show how you could **use your graph** in (ii) to find the tide height at t=10 hours.

(iv) Using $h(t) = 0.5 \sin(\pi/12\ t)$, find the two times when the tide height is +0.2 metres.

(v) If 't' is measured in hours, what are the measurement units of '$\pi/12\ t$'?

QUESTION 2 [4 + 4 + 4 + 4 + 4 = 20 marks, with bonus 2 marks]

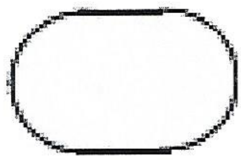
(a) Find the **first** and **second** derivatives of $y = x + 5\exp(3x)$.

(b) Find the derivative of $y = 3x \sin(4x + \pi)$.

(c) Find the slope of $y = \cosh(2x)$ at $x=2$.

(d) In question 1, the model for tide height was $h(t) = 0.5 \sin(\pi/12\ t)$. Using derivatives, at what time after t=0 is the tide falling the fastest?

(e) The diagram on the right shows a swimming pool, with end radius 'r' and straight side length 'a'. The area and perimeter of this pool is given by $A(x) = r(2a+\pi r)$ and $P(x) = 2(\pi r + a)$. [You do not have to derive these formulas.]



(i) If the perimeter P is 20m, explain how you would differentiation to find the values of 'r' and 'a' that maximise the pool area (without doing any calculations).

(ii) Bonus 2 marks. Find when the maximum area occurs (using differentiation). [There is quite a lot of work here. Attempt only if you finish the examination early.]

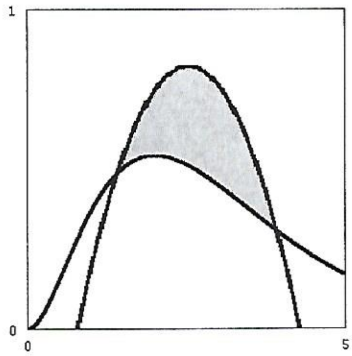
QUESTION 3 [3 + 3 + 3 + 3 + 8 = 20 marks]

(a) Find an anti-derivative of $4\sin(x) + 8x^2$.

(b) Find $\int_1^2 (3-5\exp(\frac{x}{2}))dx$.

(c) Find the indefinite integral $\int x\sin(2x)dx$.

(d) The function graphs on the right are of (A) $y = x^2\exp(-x)$ and (B) $y = -0.4x^2 + 2x - 1.6$. We wish to find the shaded area using a definite integral.



(i) Which function belongs to which curve? Why?

(ii) Before proceeding with the definite integral, two other calculations need to be made. What? [Just indicate what – do not attempt to proceed further.]

(iii) Write down the definite integral you would need to find the area. Do not attempt to evaluate the integral.

(iv) Three students attempted this problem and got the following three answers.
0.570 2.212 4.301
One answer is correct. Which one and why? [A reason is required.]

QUESTION 4 [10 + 10 = 20 marks]

(a) Consider the infinite series $\mathbf{10/2^{1.2} + 10/3^{1.2} + 10/4^{1.2} + \ldots}$

(i) If the first term of this series is t_2, (ie, $t_2 = 10/2^{1.2}$) what would be the formula for t_n?

(ii) If the first term of this series is t_0, what would be the formula for t_n?

(iii) If this series converges, what do we know about t_n (from either (i) or (ii))?

(iv) (continued from (iii)) Is this true for the series? Explain.

(v) Use the integral test to test the convergence/divergence of the series.

(b) The Taylor series for $\cosh(2x)$ is $1 + 4x^2/2! + 16x^4/4! + \ldots$

(i) Show how you would use the formula $f(x) = \Sigma f^{(n)}(x)x^n/n!$ to derive this formula.

(ii) Use the formula for cosh(2x) to find the Taylor series for sinh(2x)?

(iii) This series for cosh(2x) has an infinite "interval of convergence".
What do you understand by this?

QUESTION 5 [6 + 6 + 4 + 4= 20 marks]

(a) Find
- (i) j^3.
- (ii) $3(2 - j3) - 2(j - 1)$.
- (iii) $(j - 1)(2 + j4)$.
- (iv) $1/(j - 1)$.
- (v) the quadrants in which the answers to (ii), (iii) and (iv) lie.

(b) Convert
- (i) $6 \angle -0.7$ to rectangular form.
- (ii) $-3 + j$ to polar form.
- (iii) $6 \angle -1.6$ to exponential form.

(c) In polar form $a = 3\angle 2$, $b = 4\angle -1.2$ and $c = 2\angle (\pi/2)$
- (i) Why is 'c' a pure imaginary number?
- (ii) Find the product ab^2c (writing your answer in polar form)
- (iii) Find the quotient a/b (writing your answer in polar form)
- (iv) Use de Moivre's theorem to write down an expression for $a^{0.25}$

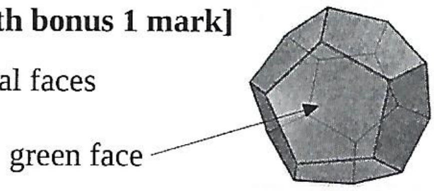
(d) The Taylor series for exp, sin and cos are:

$$\exp(x) = 1 + x^2/2! + x^3/3! + x^3/3! + \ldots$$
$$\sin(x) = x - x^3/3! + x^5/5! - x^7/7! + \ldots$$
$$\cos(x) = 1 - x^2/2! + x^4/4! - x^6/6! + \ldots$$

Use this to show that $r\exp(jx) = r\cos(x) + j\, r\sin(x)$

QUESTION 6 [3 + 3 + 4 + 3 + 1 + 1 + 3 + 2 = 20 marks, with bonus 1 mark]

A **regular dodecahedron** is a solid with 12 identical pentagonal faces (as illustrated on the right). Opposite faces are parallel.



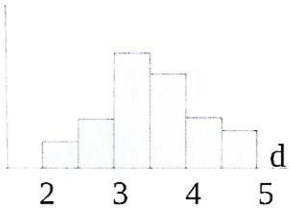
One such dodecahedron has 2 red faces, 2 green faces, and 3 blue faces. All other faces are yellow.

(a) This dodecahedron is **rolled once** and the outcome is the colour on the face ending on top. With a reason, what is the probability that:.
   (i) a **yellow** side ends on top?
   (ii) the side on top is **not blue**?

(b) In a new experiment the same dodecahedron is rolled **three times**. With a reason, what is the probability that:
   (i) all three rolls result in a **blue** face on top?
   (ii) all three rolls have the **same colour** on top?

(c) Amazingly, similar dodecahdrons start falling randomly (in time) from the sky! **On average** three fall each hour. What is the probability that in the next hour:
   (i) **exactly** three will fall?
   (ii) less than three will fall?

(d) The dodecahedrons that fall from the sky in (c) are not all the same size. A person collected and measured some. After lots had fallen he constructed a graph of their diameters (d, in cm) – and ended up with the histogram on the right:

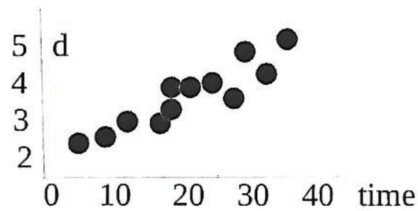


From the histogram estimate the **mean** (m) and **standard deviation** (s) of diameters of the dodecahdrons that he collected.

(e) Why from the histogram might we assume that the distribution of the diameters of falling dodecahedrons is Normal?

(f) When the formula **=norm.dist(m, s, 3, 1)** was entered into a spreadsheet (where 'm' and 's' are your numbers from (d)) the result was **0.28** Interpret this number.

(g) Another person did a different analysis of the size of the falling dodecahedrons. She plotted diameter size against time (in hours) that each fell, and ended up with the scattergram on the right (only some of the data are plotted):



Using this graph only, deduce a regression line formula that could be used to predict the size of future falling dodecahdrons.

(h) (Bonus 1 mark) We might get hit by a falling dodecahedron. Using (g), why should we **start** to get worried?

(i) What is the approximate correlation coefficient of the dodecahedron time/size data in (g)? Explain.

--------- End of Examination ---------