THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

**FIRST SEMESTER EXAMINATIONS – 2023**

**FIRST YEAR BACHELOR IN ELECTRICAL ENGINEERING,**

**FIRST YEAR BACHELOR IN MINERAL PROCESSING**

**ENGINEERING &**

**FIRST YEAR BACHELOR IN MINING ENGINEERING**

# EN112B – ENGINEERING MATHEMATICS I

**TIME ALLOWED: 3 HOURS**

**INFORMATION FOR CANDIDATES:**

1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIVE (5) questions. You are to **answer ALL** the questions.
4. All answers must be written in examination answer booklet provided. No other written materials will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.
8. A formula sheet and a normal distribution graph are attached.

**MARKING SCHEME:**

Marks are indicated at the beginning of each question. The total is **100 marks.**

**QUESTION 1 [5 + 5 + 5 = 15 marks]**

a. Find the domain and range of the following functions.

(i) $f(x) = \sin 2x$
(ii) $f(x) = 2 + \sqrt{x - 1}$

b. Find the composition of function **f o g** and **g o f** for the following functions.

(i) $f(x) = \tan x$ , $g(x) = |x + 1|$
(ii) $f(x) = 1/x$ , $g(x) = x$

c. Solve for x, if $\frac{e^x + e^{-x}}{2} = 1$.

**QUESTION 2 [10 + 5 + 5 = 20 marks]**

a. Find the derivative of the following functions with respect to x.

(i) $5x^3e^x$

(ii) $\sqrt{x} + \frac{1}{x} + \ln x$.

b. Find the maximum and minimum value of the function: $f(x) = x^3 - 6x^2 - 15x + 16$.

c. Find the equation of the tangent at point (2, 2) on the curve: $y = x^3 - x^2 + 12$.

**QUESTION 3 [5 + 5 + 10 = 20 marks]**

a. Find the four arithmetic means between 4 and 19.

b. Test the convergence of the following series: $\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \ldots + \frac{n^2}{2^n}x^5 + \ldots.$

c. Find the value of following integrals:

(i) $\int \left[x^4 + \frac{\cos x}{5} + \frac{2}{3e^{4x}}\right] dx$

(ii) $\int \frac{5x+2}{3x^2+x-4} dx$

**QUESTION 4 [10 + 7 + 8 = 25 marks]**

a. Find the value of following integrals:

(i) $\int_0^1 \frac{5}{3 + 2x^2} dx$

(ii) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \ dx$

b. Determine the coordinates of the points of the intersection of the curve $y = x^2$ and $y^2 = 8x$.

i. Sketch the curve $y = x^2$ and $y^2 = 8x$.

ii. Calculate the area enclosed between two curves.

c. Find out all roots of equation $z^3 = 1$.

**QUESTION 5 [5 + 4 + 5 + 6= 20 marks]**

a. Use de Moiver's theorem to prove the identities: $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$

b. Simplify the following expressions: $\dfrac{\cos 4\theta + i \sin 4\theta}{\cos 3\theta - i \sin 3\theta}$

c. The overall percentage of failure in a certain examination is 20. If six candidates appear in the examination, what is the probability that four students pass the examination?

d. An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that not more than two of its clients are involved in such an accident next year?

**END OF EXAM**

## Reference Material

**Composition of function:** (f o g)(x)= f(g(x))

**First principle of differentiation for** y = f(x): $\frac{dy}{dx} = \lim_{n \to 0} \frac{f(x+h)-f(x)}{h}$,

$\frac{dx^n}{dx} = nx^{n-1}$

$\frac{de^x}{dx} = e^x$

$\frac{d\cos x}{dx} = \sin x$

$\frac{d}{dx}[f(x).g(x)] = f(x)\frac{dg(x)}{dx} + g(x)\frac{d\,f(x)}{dx}$

**Maxima/Minima:** y = f(x) is maxima/minima if $\frac{d^2 y}{dx^2}$ is less than zero or greater than zero

**Tangent** equation at P = ($x_1$, $y_1$) is : $(y - y_1) = \left(\frac{dy}{dx}\right)_p (x - x_1)$

**Ratio test:** $\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \begin{pmatrix} > 0 \text{ then } u_n \text{ is divergent} \\ < 0 \text{ then } u_n \text{ is convergent} \\ = 0 \text{ then ratio test fails} \end{pmatrix}$

**Integrations**

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

$\int \sin x\,dx = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \frac{f(x)^/}{f(x)} dx = \ln f(x) + c$

$\int \frac{1}{a^2 + x^2} dx = \frac{1}{2a} \tan^{-1}\left(\frac{x}{a}\right) + c$

**Area** enclosed between two curve f(x) and g(x) over limit [a, b]: $\int_a^b [f(x) - g(x)]dx$

**Modulus** of z = x + iy = $\sqrt{x^2 + y^2}$

**Argument** of z = x + iy : $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

**Euler Formula:** $e^{i\theta} = \cos\theta + i\sin\theta$

**de Moiver's theorem:** $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

**Binomial Distribution:** P(r) = $C_r^n p^r q^{n-r}$

**Poisson Distribution:** P(r) = $\frac{e^m m^r}{r!}$, where m = np