THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS – 2022

FIRST YEAR BACHELOR IN APPLIED PHYICS

EN112 – ENGINEERING MATHEMATICS I

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIVE (5) questions. You are to **answer ALL** the questions.
4. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.
8. A formula sheet and a normal distribution graph are attached.

MARKING SCHEME

Marks are indicated at the beginning of each question. The total is **100 marks.**

QUESTION 1 [5 + 4 + 3 + 8 = 20 marks]

(a) If $f(x)=\sqrt[3]{x}\ ; x\geq 0$, then determine $f'(3)$ from the definition of derivative.

(b) Show that the function $f(x)=\begin{cases}9-2x & ;x<2\\ 1+2x & ;x\geq 2\end{cases}$

is not differentiable at $x=2$.

(c) Determine $\frac{dy}{dx}$ if $y=\left(\tan x+x^4\right)^{-6}$

(d) Water runs out at $7\,ft^3\,/\min$ from a water tank that has the shape of an inverted circular cone with base radius of 10 ft. and height 12 ft. How fast the water level in the tank will be lowering when the water will be 5 ft. deep?

QUESTION 2 [5 + 5 + 4 + 6 = 20 marks]

(a) By using the definition of definite integral evaluate $\int_0^1\left(x^3+2x^2+1\right)dx$

(b) Use the midpoint rule with $n=5$, to approximate $\int_0^2\frac{dx}{\sqrt{x}}$

(c) Integrate $\int x^2\ln x\,dx$

(d) Determine the volume of a right circular cone of radius 'r' and slant height $\sqrt{h^2+r^2}$, by using the method of surface of revolution.

QUESTION 3 [4 + 6 + 5 + 5 = 20 marks]

(a) If $z=-3-4i$, then determine $Im\left(\frac{z\bar{z}}{|z|^2}\right)$.

(b) Write the complex number $z=-\sqrt{3}+3i$ in polar form.

(c) Simplify: $(1+i)^{10}$

(d) Solve: $z^3=\sqrt{2}+\sqrt{2}i$, where z is a complex number and $i=\sqrt{-1}$.

QUESTION 4 [5 + 5 + 5 + 5 = 20 marks]

(a) Determine the median class of the following frequency distribution.

Class interval	Frequency
72-85	2
86-99	6
100-113	8
114-127	9
128-141	5
142-155	4

(b) 2% of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 150 screws chosen at random exactly 5 will be defective.

(c) As per the survey done on Covid infection in each family in a certain region the data has been arranged in the following table. Calculate χ^2 value.

Number of infected person	O_i	E_i
One	30	28
Two	25	27
Three	20	22
Four	18	14
Five	08	05
Six	04	02

(d) Determine the line of regression from the following data and hence estimate y corresponding to $x = 0.32$

x	2	4	8	10	12
y	6	5	4	2	1

QUESTION 5 [6 + 4 + 5 + (3 + 2) = 20 marks]

(a) Find the local extrema of $f(x) = x^3 - 2x^2 + 6$ by using the first derivative test.

(b) Evaluate: $\int_0^5 |x-2|dx$

(c) Express the complex number $-\sqrt{2} + \sqrt{6}i$ in exponential form.

(d) The distribution of length of a metal component produced by a machine is normal with a mean 15cm and a standard deviation of 0.5 cm. Determine the portion of components with lengths: (given: $P(0 \leq z \leq 1) = 0.3413,\ P(0 \leq z \leq 2) = 0.4772$)

(i) Greater than 16 cm.

(ii) Less than 14.5 cm.

END OF EXAM

FORMULAE

$\frac{d}{dx}\{C\}=0$; Where C is a constant.	$\int \cos x\,dx=\sin x+c$
$\frac{d}{dx}\{x^n\}=nx^{n-1}$	$\int \tan x\,dx=\ln\lvert\sec x\rvert+c$
$\frac{d}{dx}\{e^x\}=e^x$	$\int \cot x\,dx=\ln\lvert\sin x\rvert+c$
$\frac{d}{dx}\{a^x\}=a^x\ln a$	$\int \sec x\,dx=\ln\lvert\sec x+\tan x\rvert+c$
$\frac{d}{dx}\{\sin x\}=\cos x$	$\int \operatorname{cosec}x\,dx=\ln\lvert\operatorname{cosec}x-\cot x\rvert+c$
$\frac{d}{dx}\{\cos x\}=-\sin x$	$\int \frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x+c$
$\frac{d}{dx}\{\tan x\}=\sec^2 x$	$\int \frac{-1}{\sqrt{1-x^2}}dx=\cos^{-1}x+c$
$\frac{d}{dx}\{\cot x\}=-\operatorname{cosec}^2x$	$\int \frac{1}{1+x^2}dx=\tan^{-1}x+c$
$\frac{d}{dx}\{\sec x\}=\sec x.\tan x$	$\int \frac{-1}{1+x^2}dx=\cot^{-1}x+c$
$\frac{d}{dx}\{\operatorname{cosec}x\}=-\operatorname{cosec}x.\cot x$	$\int \frac{1}{x\sqrt{x^2-1}}dx=\sec^{-1}x+c$
$\frac{d}{dx}\{\ln x\}=\frac{1}{x}$	$\int \frac{-1}{x\sqrt{x^2-1}}dx=\operatorname{cosec}^{-1}x+c$
$\frac{d}{dx}\{\tan^{-1}x\}=\frac{1}{1+x^2}$	$\int_a^b f(x)dx=F(b)-F(a)$; Where $F'(x)=f(x)$
$\frac{d}{dx}\{\sin^{-1}x\}=\frac{1}{\sqrt{1-x^2}}$	$\int_a^b f(x)dx=\lim\limits_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n}f\left(a+\frac{i(b-a)}{n}\right)$
$\int x^n dx=\frac{x^{n+1}}{n+1}+c;\quad n\neq-1$	$\int_a^b f(x)dx=\sum_{i=1}^{n}f(\overline{x_i})\Delta x;\ \Delta x=\frac{b-a}{n},\ \overline{x_i}=\frac{x_{i-1}+x_i}{2}$
$\int e^x dx=e^x+c$	$\int(u.v)dx=u\int v dx-\int\left\{\frac{d(u)}{dx}\int v dx\right\}dx+c$
$\int a^x dx=\frac{a^x}{\ln a}+c$	$\sin^2x+\cos^2x=\sec^2x-\tan^2x=\operatorname{cosec}^2x-\cot^2x=1$
$\int \frac{1}{x}dx=\ln x+c$	$e^{i\theta}=\cos\theta+i\sin\theta$
$\int \sin x\,dx=-\cos x+c$	Normal equations of $y=a+bx$ are $\sum y=na+b\sum x,\quad \sum xy=a\sum x+b\sum x^2$