THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS 2021

FIRST YEAR APPLIED PHYSICS
FIRST YEAR BIOMEDICAL ENGINEERING
FIRST YEAR ELECTRICAL ENGINEERING
FIRST YEAR MINING ENGINEERING
FIRST YEAR MINERAL PROCESSING

EN112 - ENGINEERING MATHEMATICS 1 (AP,BE,EE,MinE,MP)

TIME ALLOWED – 3 HOURS

Information for Candidates

1. Write your name, student number, and program of study clearly on the front page of your answer booklet. Do it **now.**
2. You have 10 minutes to read this examination paper. During this time you must **NOT** write **inside** your answer booklet. You can make notes on the examination paper.
3. A scientific calculator is permitted, though you do not have to use one.
   Other electronic devices are not permitted. Notes and headphones are not permitted.
4. At the conclusion of the examination you must **immediately** put your pens down. You are **NOT** permitted to write inside your answer booklet after the "end of examination" announcement.
5. You can answer the questions in any order. Start each question on a new page. After you have finished the exam, indicate the order in which you answered questions in the left column of the marks box on the cover of the answer booklet.
6. **Do NOT check your answers to any question unless specifically asked to do so.**
   Do NOT simplify answers unless specifically asked to do so.
7. There are 5 questions. Marks for each question and question part are shown at the top of each question. You should attempt all questions.

QUESTION 1 [3 + 4 + 8 = 15 marks]

(a) What is the (natural) domain and range of $y = 3\sqrt{x-1}+2$?

(b) The **sound density** x metres from a source is modelled by the function $D(x) = ax^{-2}$, where D is measured in decibels (db) and x is in metres (m).

(i) What type (ie, classification) of function is D(x) ?

(ii) The sound density 5 metres from a source is 10dB. What is the density 10 metres from the source?

(c) A liquid cools according to the formula ***T(t) = 65 exp(–t/10) + 20***, where T is degrees Celcius, and t is the recording time in minutes.

(i) What is the temperature at time 0?

(ii) What would the temperature be after a very long period?

(iii) When will the temperature be 50°C? [An expression for t will be sufficient.]

(iv) Show the shape of the graph of T(t).

QUESTION 2 [2 + 3 + 3 + 3 + 3 + 1 = 15 marks]

This question uses the sinusoidal function ***y(x) = 10 sin(4x)***, which has a period of π/2.

(a) Write down two functions *f(x)* and *g(x)* such that *f(g(x)) = 10 sin(4x).*

(b) Find the inverse of the function *y = 10 sin(4x)*, and include **any restrictions** on the inverse.

(c) Sketch the first **full period** of *y = 10 sin(4x)* to the **right** of the y-axis.

(d) Show on your graph the **two** solutions of y(x) = 5.

(e) Use your inverse function in (b), and your sketch in (c) to deduce one of the solutions in (d). [An expression for the solution will suffice.]

(f) **How** would you check that your solution in (e) is correct (don't do it!).

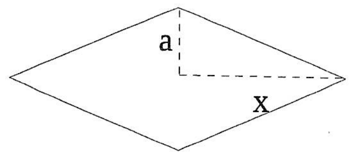
QUESTION 3 [3 + 3 + 2 +7 = 15 marks + bonus 3 marks]

(a) Find the **first and second** derivatives of $y = 5\ln(x) + 3$.

(b) Find the derivative of $y = 3x\cos(x^2)$.

(c) Find the slope of $y = 0.25x^2 - 2x$ at $x=1$.

(d) The diagram on the right shows a diamond shape, where **a** and **x** are the height and width of ¼ of the diamond. The area of the diamond is fixed at **8 cm²**.



(i) Show that $a = 4/x$.

(ii) Using (i), show with reasons that the diamond's perimeter P(x) is $4\sqrt{x^2+\frac{16}{x^2}}$.

(iii) In terms of derivatives, what must be true at the value of x for which the perimeter is minimum?

(iv) Find P'(x).

[Bonus 3 marks, to try if you finish the exam early]
Using (iii) and (iv), find the value of x that minimises the diamond's perimeter.]

QUESTION 4 [3 + 3 + 6 + 3 = 15 marks]

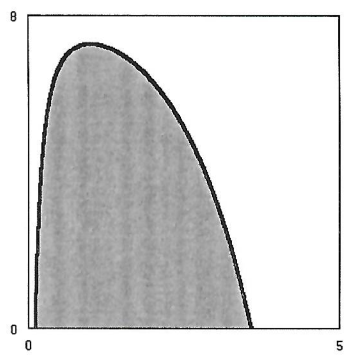
(a) Find an anti-derivative of $1+\frac{1}{x}-\frac{1}{x^2}$.

(b) Find $\int_0^{2\pi}(3-5\sin(\frac{x}{2}))dx$.

(c) Find the indefinite integrals (i) $\int x\exp(x^2)\,dx$.

(ii) $\int x\exp(x)\,dx$.

(d) This question part involves integrals, but requires **no calculations** and **no integrations**. It involves the area under the curve on the right, which is $y = 10 - \exp(x)/x$, between x=0.112 and x=3.577. [The base is the x-axis.]



(i) What is the approximate shaded area (in square units)? [A single number estimate to one significant figure is required! No calculations required.]

(ii) How you would find the exact shaded area? [You will require a formula, but no calculations.]

QUESTION 5 [6 + 6 + 4 + 4= 20 marks]

This question concerns complex numbers in rectangular, polar and exponential form.

(a) If $a = 2 + j3$ and $b = j - 1$ find (in simplified rectangular form)
(i) $3a - 2b + j$.
(ii) ab.
(iii) 1/a.

(b) Convert
(i) $-3 + j2$ to polar form.
(ii) $4 \angle 1.2$ to rectangular form.
(iii) $4 \angle 1.2$ to exponential form.

(c) In polar form $a = 3\angle 2$, $b = 4\angle -1.2$ and $c = 2\angle \pi/2$

(i) What is special about the number c?
(ii) Write down the value of ab/c in polar form.
(iii) Write down the value of a^4 in polar form.

(d) Find the three cube roots of $4 \angle 2.1$ (leaving your answer in polar form).

QUESTION 6 [6 + 4 + 4 + 6 = 20 marks]

(a) Consider the **infinite series** $S = 1/2^1 - 1/3^2 + 1/4^3 - 1/5^4 \ldots$

(i) What will be the next term of this series?
(ii) If the first term is **term 0**, what is a formula for term n?
(iii) How do we know that this series converges.
(iv) The partial sums S_4 and S_5 are (respectively) 0.4029 and 0.4030, what can we infer about the sum S?

(b) Use the integral test to test the convergence/divergence of $1 + 1/2 + 1/3 + 1/4 + 1/5 + \ldots$

(c) The n^{th} term of the power series $S(x) = (x/2)^2+(x/2)^3+(x/2)^4+(x/2)^5+ \ldots$ is $t_n = (x/2)^{n+2}$ [don't show this]. Use this t_n and the ratio test formula $r = \lim_{x\to\infty} \left|\frac{t_{n+1}}{t_n}\right|$ to find the radius or interval of convergence of the power series.

(d) Using the formula $f(x) = f(0) + f'(0)x + f''(0)x^2/2! + f'''(0)x^3/3! + \ldots$
find the Taylor Series for $y = \sin(x)$.

--------- End of Examination ---------