THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS – 2022

FIRST YEAR MECHANICAL ENGINEERING & CIVIL ENGINEEING

EN121(A) – ENGINEERING MATHEMATICS II

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklet/s.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains five (5) questions. You should attempt all the questions.
4. Make sure you have 6 pages.
5. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
6. Start the answer for each question on a new page.
7. Do not use red ink or pencil.
8. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
9. Scientific and business calculators are allowed in the examination room.
10. The last three pages contains a formula sheet for students information.

MARKING SCHEME

Marks are indicated at the beginning of each question. Total mark is 100.

Question 1 (5 + 5 +15 = 25 Marks)

(a) If $A = \begin{bmatrix} -1 & 1 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix}$, Find matrix C, if C = 2A + 3B.

(b) Find the value of determinant of matrix A if $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

(c) Solve the following system of linear equations by Gauss – Jordan Elimination method.

$x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 2$

Question 2 (5 + 5 + 10 = 20 Marks)

(a) Find the length of vector $|r|$ for $\bar{r} = \hat{\imath} + 4\hat{\jmath} - 2\hat{k}$.

(b) Find the value of the given dot product $(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) \bullet (2\hat{\imath} - 2\hat{\jmath} - 3\hat{k})$.

(c) Find the volume of the parallelepiped formed by three adjacent vectors

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$, $\bar{b} = -3\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$, and $\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$.

Question 3 (5 + 5 + 10 = 20 Marks)

(a) Solve the Differential Equation, explaining what makes them separable. $\frac{dy}{dx} + \frac{2x}{1+y^2} = 0$.

(b) Solve the Differential Equation, by suitable method $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$.

(c) Solve the linear Differential Equation, explaining what makes them linear: $\frac{dy}{dx} + xy = x^3$.

Question 4 (5 + 15 = 20 Marks)

(a) Solve the Second Order Linear Differential Equation $\frac{d^2y}{dx^2} - 9y = 0$.

(b) Solve Second Order Differential Equation by finding the general solution of a related homogeneous equation and a particular solution of the non-homogeneous equation.

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x} + \sin 2x$

Question 5 (3 + 4 + 8 = 15 Marks)

(a) Find the LT of the given functions: $f(t) = e^{3t} . \cos(2t)$.

(b) Find the Laplace Inverse of the function $F(s) = \frac{1}{4s^2-25} + \frac{1}{s^3}$.

(c) Solve the differential equations using Laplace Transforms

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0, \text{ while } y = 0 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 0.$$

END OF EXAMINATION

Reference Material

Determinant

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$ Expanding |A| along C_1, we get

$$|A| = a_{11}(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}(a_{22}\,a_{33} - a_{23}\,a_{32}) - a_{21}(a_{12}\,a_{33} - a_{13}\,a_{32}) + a_{31}(a_{12}\,a_{23} - a_{13}\,a_{22})$$

Elementary Transformations:: When we apply elementary operation then value of matrix will not changes. There are three type of Elementary Operations/Transformations

1. R_{ij} or C_{ij} : Two Rows/Column may be switched.
2. aR_i or aC_i: Any Row/Column may be multiplied by a number.
3. $R_i \to aR_j + R_i$ or $C_i \to aC_j + C_i$: At any time replace a row with a multiple of **another row** added to the row.

Echelon Form

Echelon form. Row reduced form or Upper triangular form

Procedure: Apply Elementary transformation and increase zero on left hand side of the matrix when we move towards down

Gauss Jordan method: Method to find out value of variable in system of equation

$a_1 x + b_1 y + c_1 z = d_1$
$a_2 x + b_2 y + c_2 z = d_2$
$a_3 x + b_3 y + c_3 z = d_3$

System of equation converted in Augmented matrix

$$(\mathbf{A} \,|\, \mathbf{B}) = \left(\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right)$$

Apply elementary transformation and convert Augmented matric in Normal form

$$(\mathbf{A} \,|\, \mathbf{B}) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & p \\ 0 & 1 & 0 & r \\ 0 & 0 & 1 & m \end{array}\right)$$

Gives the value x = p, y = r and z = m

Product of vectors

Dot product: $(a\hat{i} + b\hat{j} + c\hat{k}) \bullet (p\hat{i} + q\hat{j} + r\hat{k}) = a \times p\,(\hat{i} \bullet \hat{i}) + b \times q\,(\hat{j} \bullet \hat{j}) + c \times r\,(\hat{k} \bullet \hat{k})$
and $(\hat{i} \bullet \hat{i}) = (\hat{j} \bullet \hat{j}) = (\hat{k} \bullet \hat{k}) = 1$

Cross products: $(a\hat{i} + b\hat{j} + c\hat{k}) \times (p\hat{i} + q\hat{j} + r\hat{k}) = \begin{bmatrix} i & j & k \\ a & b & c \\ p & q & r \end{bmatrix}$

Volume of Parallelogram formed by three vectors $\bar{r_1} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\bar{r_2} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, $\bar{r_3} = a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$, is $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

Linear Differential Equations, Bernoulli

Its general form is: $\frac{dy}{dx} + P(x)\, y = Q(x)$

To solve such equation

Step 1 Identify P (x) and Q(x)

Step 2 Find **Integrating Factor: I.F. =** $e^{\int P(x)\, dx}$

Step 3 Solution of given DE is $y.\mathbf{I.F.} = \int \mathbf{Q.I.F.\,dx + c}$

Second Oder Differential Equations

General Solution = Complementary Function + Particular Solution

Complementary Function:

To find out complementary function first we make homogeneous equation to the given second order Differential Equation by equating left side to zero then substitute $y = e^{\lambda x}$ and get characteristic equation solve it

The Characteristic Equation (CE)

From a homogeneous linear 2nd order DE with constant coefficients after substituting $y = e^{\lambda x}$ we get algebraic equation $\lambda^2 + a\lambda + b = 0$. Solve it to get value of λ as λ_1 and λ_2

$\lambda_1, \lambda_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Case 1 $\lambda_1 \neq \lambda_2$, and are real then the DE solutionis $\mathbf{y(x) = A\, e^{\lambda_1 x} + B e^{\lambda_2 x}}$

Case 2 λ_1, λ_2, are complex number = $\alpha \pm i\beta$

then the DE solutionis $\mathbf{y(x)} = e^{\alpha x}(\mathbf{A \cos \beta x + B \sin \beta x})$

Particular Integral:

Particular Integral is solution of the actual non-homogeneous DE

For second order Differential Equation $(D^2 + a D + b)\, y = f(x)$

$y_{\text{Particular Integral}} = \frac{1}{D^2 + aD + b} f(x)$

Case 1: $f(x) = e^x$; $y_{\text{Particular Integral}} = \frac{1}{F(D)} e^{ax} = \frac{1}{F(a)} e^x$

Case 2: $f(x) = \sin ax$; $y_{\text{Particular Integral}} = \frac{1}{F(D^2)} \sin ax = \frac{1}{F(-(a^2))} \sin ax$

Laplace Transformation

1. $L(1)=\frac{1}{s}$
2. $L\left(t^n\right)=\frac{n!}{s^{n+1}}$, when $n=0, 1, 2, 3...$
3. $L\left(e^{at}\right)=\frac{1}{s-a}$ $(s>a)$
4. $L(\cos at)=\frac{s}{s^2+a^2}$ $(s>0)$
5. $L(\sinh at)=\frac{a}{s^2-a^2}$ $(s^2>a^2)$
6. $L(\sin at)=\frac{a}{s^2+a^2}$ $(s>0)$

Properties of Laplace Transform

FIRST SHIFTING PROPERTY If $L\{f(t)\}=F(s)$, then $L[e^{at}f(t)]=F(s-a)$

LAPLACE TRANSFORM OF *t.f(t)* (Multiplication by *t*)

If $L[f(t)]=F(s)$, then $L[t^n f(t)]=(-1)^n\frac{d^n}{ds^n}[F(s)]$.

Laplace Inverse Transform

1. $L^{-1}\left(\frac{1}{s}\right)=1$
2. $L^{-1}\frac{1}{s^n}=\frac{t^{n-1}}{(n-1)!}$
3. $L^{-1}\frac{1}{s-a}=e^{at}$
4. $L^{-1}\frac{s}{s^2-a^2}=\cosh at$
5. $L^{-1}\frac{1}{s^2-a^2}=\frac{1}{a}\sinh at$
6. $L^{-1}\frac{1}{s^2+a^2}=\frac{1}{a}\sin at$
7. $L^{-1}\frac{s}{s^2+a^2}=\cos at$
8. $L^{-1}F(s-a)=e^{at}f(t)$
9. $L^{-1}\frac{1}{(s-a)^2+b^2}=\frac{1}{b}e^{at}\sin bt$
10. $L^{-1}\frac{s-a}{(s-a)^2+b^2}=e^{at}\cos bt$

LAPLACE TRANSFORM OF THE DERIVATIVE OF *f(t)*

$L[f'(t)]=sL[f(t)]-f(0)$ where $L[f(t)]=F(s)$.

$L[f''(t)]=s^2F(s)-sf(0)-f'(0)$

Solution of Differential Equation by Laplace Transform:

Step 1. Apply Laplace transform on the Differential Equation
Step 2. Simplify algebraic Equation
Step 3. Apply Laplace Inverse Transform to get the solution