THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATION - FEBRUARY 2022

FIRST YEAR APPLIED PHYSICS AND BIOMEDICAL ENGINEERING

EN121C - ENGINEERING MATHEMATICS II

SUPPLEMENTARY EXAM

TIME ALLOWED: 3 HOURS

- **INFORMATION FOR CANDIDATES**

1. Write your name and student number clearly on the front of the examination answer booklets.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains **five(5)** questions. You must attempt **all**. The questions can be done in any order but all parts to the same question must be kept together.
4. Show all working.
5. All answers must be written in examination answer booklet(s) provided. No other written materials will be accepted.
6. Start the answer for each question on a new page. Do not use red ink.
7. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
8. Scientific and business calculators are allowed in the examination room.
9. The last three pages contains information sheet for student use.

- **MARKING SCHEME**

Marks are indicated in brackets for each question. Total is **100 marks** with 100% weight. To pass this course you have to score 50 and above.

Question 1 [16 marks]

Solve the initial value problem $x^2y'' - 9xy' + 24y = 0$ with $y(1) = 1$ and $y'(1) = 10$.

Question 2 [22 marks]

Use the Gauss-Jordan method to solve the following linear system:

$$\begin{cases} \frac{1}{4}x_1 + \frac{1}{5}x_2 + \frac{1}{6}x_3 = 9 \\ \frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 = 8 \\ \frac{1}{2}x_1 + x_2 + 2x_3 = 8 \end{cases}$$

Question 3 [19+9=28 marks]

(a) Use the Laplace transform to solve the initial value problem $y'' - 4y' + 4y = \cos(t)$ with $y(0) = 1$ and $y'(0) = -1$,

(b) Solve the following integral equation:

$$f(t) = -t + \int_0^t f(t-\tau)\sin(\tau)d\tau.$$

Question 4 [7+7+10=24 marks]

Consider the differential equation $y + xy' = 0$ with $x > 0$:

(a) Solve it as a separable differential equation,

(b) Solve it as a linear differential equation $\left(y' + p(x)y = q(x)\right)$,

(c) Solve it as a homogeneous differential equation $\left(y' = f\left(\frac{y}{x}\right)\right)$.

The asnwers for the cases (a), (b) and (c) should be exactly same.

Question 5 [5+5=10 marks]

(a) Let F and G be nonparallel vectors and let R be the parallelogram formed by representing these vectors as arrows from a common point. Find that the area of this parallelogram using cross product,

(b) Find a vector that is orthogonal to the plane containing the points $P(3, 0, 1)$, $Q(4, -2, 1)$ and $R(5, 3, -1)$.

End of Exam

Reference Material

(1) Consider the following 3×3 augmented matrix:

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right].$$

To solve the corresponding system of equations related to this matrix by Gauss-Jordan method, use the legal row operations on augmented matrix to convert coefficient matrix into an identity matrix. On the other hand, we have to

- Convert a_{11} into 1,
- Convert a_{21} into 0,
- Convert a_{22} into 1,
- Convert a_{31} into 0,
- Convert a_{32} into 0,
- Convert a_{33} into 1,
- Convert a_{12} into 0,
- Convert a_{13} into 0,
- Convert a_{23} into 0.

by using legal row operations.

(2) How to solve Euler's differential equation:

i. Arrange the differential equation into standard form $x^2y'' + Axy' + By = 0$,

ii. Solve the differential equation $u'' + (A-1)u' + Bu = 0$ for $u(\mathbf{t})$,

iii. Find $y(x)$ using the following equation $y(x) = u(\ln(x))$.

(3) Consider the linear second order differential equation $ay'' + by' + cy = 0$ where a, b and c are constant. The equation $a\lambda^2 + b\lambda + c = 0$ is called the characteristic equation for this differential equation. Characteristic equation has two solutions in the following form:

$$\lambda_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \qquad \lambda_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

i. When $b^2 - 4ac > 0$, general solution of differential equation can be found by

$$y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t},$$

ii. When $b^2 - 4ac = 0$, then $\lambda_1 = \lambda_2 = \lambda$ and the general solution of differential equation can be found by

$$y(t) = (c_1 + c_2 t)e^{\lambda t},$$

iii. When $b^2 - 4ac < 0$, then $\lambda_1 = \alpha + i\beta$ and $\lambda_2 = \alpha - i\beta$. Then the general solution of differential equation can be found by

$$y(t) = c_1 e^{\alpha t}\cos(\beta t) + c_2 e^{\alpha t}\sin(\beta t).$$

(4) Partial Fraction Decomposition

Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\dfrac{A}{ax+b}$
$(ax+b)^k$	$\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\cdots+\dfrac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\dfrac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\dfrac{A_kx+B_k}{(ax^2+bx+c)^k}$

(5) Laplace Inverse Transforms of Selected Functions

$F(s)$	$\mathscr{L}^{-1}\{F(s)\}$	$F(S)$	$\mathscr{L}^{-1}\{F(s)\}$	$F(S)$	$\mathscr{L}^{-1}\{F(s)\}$
$\dfrac{1}{s}$	1	$\dfrac{n!}{s^{n+1}}$	t^n	$\dfrac{s^2-a^2}{(s^2+a^2)^2}$	$t\cos(at)$
$\dfrac{1}{s-a}$	e^{at}	$\dfrac{n!}{(s-a)^{n+1}}$	$t^n e^{at}$	$\dfrac{b}{(s-a)^2+b^2}$	$e^{at}\sin(bt)$
$\dfrac{a-b}{(s-a)(s-b)}$	$e^{at}-e^{bt}$	$\dfrac{a}{s^2+a^2}$	$\sin(at)$	$\dfrac{s-a}{(s-a)^2+b^2}$	$e^{at}\cos(bt)$
$\dfrac{s}{s^2+a^2}$	$\cos(at)$	$\dfrac{2as}{(s^2+a^2)^2}$	$t\sin(at)$	$\dfrac{a}{s^2-a^2}$	$\sinh(at)$
$\dfrac{s}{s^2-a^2}$	$\cosh(at)$	e^{-as}	$\delta(t-a)$		

(6) The Convolution Theorem

$$\begin{aligned}\mathscr{L}\{f(t)*g(t)\} &= \mathscr{L}\{f(t)\}\mathscr{L}\{g(t)\}\\ \mathscr{L}^{-1}\{F(s)G(s)\} &= \mathscr{L}^{-1}\{F(s)\}*\mathscr{L}^{-1}\{G(s)\}\end{aligned}$$

(7) Transform of a Higher Derivative for $n = 1$ and $n = 2$

$$\begin{aligned} \mathscr{L}\{y'\} &= s\mathscr{L}\{y\} - y(0) \\ \mathscr{L}\{y''\} &= s^2\mathscr{L}\{y\} - sy(0) - y'(0) \end{aligned}$$

(8) Steps to solve a separable ODE:

i. Rewrite the ODE in the following form

$$\frac{dy}{G(y)} = F(x)dx,$$

ii. Integrate both side of the first step,

$$\int \frac{dy}{G(y)} = \int F(x)dx,$$

iii. Try to find the resulting equation for y in terms of x. Some times it is not possible to do it, then the solution will be in implicit form,

iv. The solution of the equation $G(y) = 0$ will be also a solution for ODE, check it also.

(9) Steps to solve linear ODEs $y' + p(x)y = q(x)$:

i. Find $g(x) = e^{\int p(x)dx}$,

ii. Then the final solution of ODE for any $c \in R$ can be found by

$$y = \left(\frac{1}{g(x)}\right)\left(\int q(x)g(x)dx\right) + \frac{c}{g(x)}.$$

(10) Steps to solve differential equation $y' = f(\frac{y}{x})$:

i. Arrange the differential equation into standard form $y' = f(\frac{y}{x})$,

ii. Replace $\frac{y}{x}$ with u,

iii. Replace y' with $u'x + u$,

iv. Replace u' with $\frac{du}{dx}$ and find a separable differential equation in terms of u and x,

v. Solve the obtained separable differential equation for u,

vi. Replace back u with $\frac{y}{x}$ and solve for y.

(11)

i. If F and G are nonzero vectors, then $F \times G = 0$ if and only if F and G are parallel,

ii. If F and G are nonzero vectors, then $F.G = 0$ if and only if F and G are orthogonal,

iii. Three vectors $\vec{u}, \vec{v}$ and $\vec{w}$ are in the same plane if and only if $\vec{u}.(\vec{v} \times \vec{w}) = 0$.