# THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY
## DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE
## SECOND SEMESTER EXAMINATION - OCTOBER 2022
## FIRST YEAR APPLIED PHYSIC & BIOMEDICAL ENGINEERING
## EN121C - ENGINEERING MATHEMATICS II
## TIME ALLOWED: 3 HOURS

- **INFORMATION FOR CANDIDATES**

1. Write your name and student number clearly on the front of the examination answer booklets.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains **six(6)** questions. You must attempt **all**. The questions can be done in any order but all parts to the same question must be kept together.
4. Show all working.
5. All answers must be written in examination answer booklet(s) provided. No other written materials will be accepted.
6. Start the answer for each question on a new page. Do not use red ink.
7. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
8. Scientific and business calculators are allowed in the examination room.
9. The last three pages contains information sheet for student use.

- **MARKING SCHEME**

Marks are indicated in brackets for each question. Total is **100 marks** with 50% weight.

## Question 1 [15+5+3=23 marks]

**(a)** Use the Gauss-Jordan method to solve the following linear system of equations:

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 29 \\ -x_1 + 3x_2 - 7x_3 = -69 \\ 8x_1 + x_2 + x_3 = 2 \end{cases}$$

**(b)** Find the value of $X$ that make the following matrix singular:

$$\begin{bmatrix} 16 & 17 & 18 & 19 & 0 & 29 \\ 21 & 22 & 23 & 24 & 25 & 30 \\ 1 & 0 & 3 & 0 & 0 & 0 \\ 6 & 0 & 8 & 9 & 0 & 27 \\ 11 & 0 & 13 & 0 & 0 & 28 \\ 31 & 0 & X & 0 & 0 & 0 \end{bmatrix}$$

**(c)** Let $F$ and $G$ be nonparallel vectors and let $R$ be the parallelogram formed by representing these vectors as arrows from a common point. Show that the area of this parallelogram is $||F \times G||$.

## Question 2 [10 marks]

Determine $\alpha$ so that the differential equation is an exact differential equation. Obtain the general solution.

$$3x^2 + xy^\alpha - x^2y^{\alpha-1}y' = 0.$$

## Question 3 [10+9=19 marks]

Consider the following differential equation

$$y'' - 3y' = 2e^{2x}\sin(x),$$

**(a)** Find the general solution using the method of undetermined coefficients,

**(b)** Find the general solution using the method of variation of parameters,

**YOUR ANSWERS FOR BOTH CASES SHOULD BE SAME.**

## Question 4 [7+8=15 marks]

Find the general solution:

**(a)** $y' = \dfrac{y}{x + y^3}$,

**(b)** $xy' = \dfrac{y}{\ln(y) - \ln(x)} + y$.

## Question 5 [4+4=8 marks]

Use table to determine the inverse Laplace transform of the function:

**(a)** $F(s) = \dfrac{2}{7s^2 - 9}$.

**(b)** $G(s) = \dfrac{-5s}{(4s^2 + 1)^2}$.

## Question 6 [10+15=25 marks]

**(a)** Use the convolution theorem to write a formula for the solution in terms of $f$:

$$y'' - 25y = f(t) \quad \text{with} \quad y(0) = 2 \quad \text{and} \quad y'(0) = -4,$$

**(b)** Use the Laplace transform to solve the system of initial value problems:

$$\begin{cases} x' + y' + x - y = 0 \\ x' + 2y' + x = 1 \end{cases} \quad \text{with} \quad x(0) = y(0) = 0.$$

**End of Exam**

# Reference Material

**(1)** Consider the following $3 \times 3$ augmented matrix:

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right].$$

To solve the corresponding system of equations related to this matrix by Gauss-Jordan method, use the legal row operations on augmented matrix to convert coefficient matrix into an identity matrix. On the other hand, we have to

- Convert $a_{11}$ into 1,
- Convert $a_{21}$ into 0,
- Convert $a_{22}$ into 1,
- Convert $a_{31}$ into 0,
- Convert $a_{32}$ into 0,
- Convert $a_{33}$ into 1,
- Convert $a_{12}$ into 0,
- Convert $a_{13}$ into 0,
- Convert $a_{23}$ into 0.

by using legal row operations.

**(3)** The differential equation $M(x,y)dx + N(x,y)dy = 0$ is exact if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

**(4)** Steps to find potential function:

i. Compute $\displaystyle\int M(x,y)dx$,

ii. Plug obtained value in the step 1 into $\phi(x,y) = \displaystyle\int M(x,y)dx + g(y)$,

iii. Find $\dfrac{\partial \phi}{\partial y}$ from step 2,

iv. Solve $\dfrac{\partial \phi}{\partial y} = N(x,y)$ for $g'(y)$,

v. Integrate $g'(y)$ respect to $y$ to find $g(y)$,

vi. Plug obtained $g(y)$ found in step 4 into the step 2 and so $\phi(x,y)$ will be found.

## (5) Partial Fraction Decomposition

| Factor in denominator | Term in partial fraction decomposition |
|---|---|
| $ax+b$ | $\dfrac{A}{ax+b}$ |
| $(ax+b)^k$ | $\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\cdots+\dfrac{A_k}{(ax+b)^k}$ |
| $ax^2+bx+c$ | $\dfrac{Ax+B}{ax^2+bx+c}$ |
| $(ax^2+bx+c)^k$ | $\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\dfrac{A_kx+B_k}{(ax^2+bx+c)^k}$ |

## (6) Laplace Inverse Transforms of Selected Functions

| $F(s)$ | $\mathscr{L}^{-1}\{F(s)\}$ | $F(S)$ | $\mathscr{L}^{-1}\{F(s)\}$ | $F(S)$ | $\mathscr{L}^{-1}\{F(s)\}$ |
|---|---|---|---|---|---|
| $\dfrac{1}{s}$ | $1$ | $\dfrac{n!}{s^{n+1}}$ | $t^n$ | $\dfrac{s^2-a^2}{(s^2+a^2)^2}$ | $t\cos(at)$ |
| $\dfrac{1}{s-a}$ | $e^{at}$ | $\dfrac{n!}{(s-a)^{n+1}}$ | $t^n e^{at}$ | $\dfrac{b}{(s-a)^2+b^2}$ | $e^{at}\sin(bt)$ |
| $\dfrac{a-b}{(s-a)(s-b)}$ | $e^{at}-e^{bt}$ | $\dfrac{a}{s^2+a^2}$ | $\sin(at)$ | $\dfrac{s-a}{(s-a)^2+b^2}$ | $e^{at}\cos(bt)$ |
| $\dfrac{s}{s^2+a^2}$ | $\cos(at)$ | $\dfrac{2as}{(s^2+a^2)^2}$ | $t\sin(at)$ | $\dfrac{a}{s^2-a^2}$ | $\sinh(at)$ |
| $\dfrac{s}{s^2-a^2}$ | $\cosh(at)$ | $e^{-as}$ | $\delta(t-a)$ | | |

## (7) The Convolution Theorem

$$\begin{aligned}\mathscr{L}\{f(t)*g(t)\} &= \mathscr{L}\{f(t)\}\mathscr{L}\{g(t)\}\\ \mathscr{L}^{-1}\{F(s)G(s)\} &= \mathscr{L}^{-1}\{F(s)\}*\mathscr{L}^{-1}\{G(s)\}\end{aligned}$$

**(8)** Transform of a Higher Derivative for $n = 1$ and $n = 2$

$$\begin{aligned}\mathscr{L}\{y'\} &= s\mathscr{L}\{y\} - y(0)\\ \mathscr{L}\{y''\} &= s^2\mathscr{L}\{y\} - sy(0) - y'(0)\end{aligned}$$

**(9)** Steps to solve a separable ODE:

i. Rewrite the ODE in the following form

$$\frac{dy}{G(y)} = F(x)dx,$$

ii. Integrate both side of the first step,

$$\int \frac{dy}{G(y)} = \int F(x)dx,$$

iii. Try to find the resulting equation for $y$ in terms of $x$. Some times it is not possible to do it, then the solution will be in implicit form,

iv. The solution of the equation $G(y) = 0$ will be also a solution for ODE, check it also.

**(10)** Steps to solve linear ODEs $y' + p(x)y = q(x)$:

i. Find $g(x) = e^{\int p(x)dx}$,

ii. Then the final solution of ODE for any $c \in R$ can be found by

$$y = \left(\frac{1}{g(x)}\right)\left(\int q(x)g(x)dx\right) + \frac{c}{g(x)}.$$

**(11)** Steps to solve differential equation $y' = f(\frac{y}{x})$:

i. Arrange the differential equation into standard form $y' = f(\frac{y}{x})$,

ii. Replace $\frac{y}{x}$ with $u$,

iii. Replace $y'$ with $u'x + u$,

iv. Replace $u'$ with $\frac{du}{dx}$ and find a separable differential equation in terms of $u$ and $x$,

v. Solve the obtained separable differential equation for $u$,

vi. Replace back $u$ with $\frac{y}{x}$ and solve for $y$.

**(12)** i. If $F$ and $G$ are nonzero vectors, then $F \times G = 0$ if and only if $F$ and $G$ are parallel,

ii. If $F$ and $G$ are nonzero vectors, then $F.G = 0$ if and only if $F$ and $G$ are orthogonal,

iii. Three vectors $\vec{u}, \vec{v}$ and $\vec{w}$ are in the same plane if and only if $\vec{u}.(\vec{v} \times \vec{w}) = 0$.

**(13)** The Wronskian of two functions $y_1(x)$ and $y_2(x)$ will be defined as the following

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2$$

**(14)** To find the particular solution of the differential equation $ay'' + by' + cy = f(x)$ using method of variation of parameters, follow these steps:

i. Find the value of $y_1$ and $y_2$ by solving the homogeneous problem $ay''+by'+cy = 0$,

ii. Calculate the $W(y_1, y_2)$,

iii. Compute $u_1(x) = -\int \frac{y_2(x)f(x)}{W(y_1, y_2)}dx$,

iv. Compute $u_2(x) = \int \frac{y_1(x)f(x)}{W(y_1, y_2)}dx$,

v. Then the particular solution will be found by $Y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$.

**(15)** To find the particular solution of the differential equation $ay'' + by' + cy = f(x)$ using method of undetermined coefficients, use the following table:

| $f(x)$ | $Y_p(x)$ |
|---|---|
| $P(x)$ | $Q(x)$ |
| $Ae^{cx}$ | $Re^{cx}$ |
| $A\cos(\beta x)$ | $C\cos(\beta x) + D\sin(\beta x)$ |
| $A\sin(\beta x)$ | $C\cos(\beta x) + D\sin(\beta x)$ |
| $P(x)e^{cx}$ | $Q(x)e^{cx}$ |
| $P(x)\cos(\beta x)$ | $Q(x)\cos(\beta x) + R(x)\sin(\beta x)$ |
| $P(x)\sin(\beta x)$ | $Q(x)\cos(\beta x) + R(x)\sin(\beta x)$ |
| $P(x)e^{cx}\cos(\beta x)$ | $Q(x)e^{cx}\cos(\beta x) + R(x)e^{cx}\sin(\beta x)$ |
| $P(x)e^{cx}\sin(\beta x)$ | $Q(x)e^{cx}\cos(\beta x) + R(x)e^{cx}\sin(\beta x)$ |