PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS - 2021

FIRST YEAR BACHELOR OF CIVIL ENGINEERING
FIRST YEAR BACHELOR OF MECHANICAL ENGINEERING

EN121 – ENGINEERING MATHEMATICS 2

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1 You have 10 minutes to read this paper. You must not begin writing during this time.
2 Write your name and student number clearly on the front of the examination answer booklet.
3 There are 8 questions. You should attempt ALL questions.
4 All answers must be written in the examination answer booklets provided. No other written material will be accepted.
5 Start the answer for each question on a new page. Do not use red ink or pencil.
6 Notes and textbooks are not allowed in the examination room.
7 Mobile phones and other recording devices are not allowed in the examination room.
8. A tables of **Laplace Transforms** is attached at the end of the questions.

MARKING SCHEME

Marks are indicated at the beginning of each question. The total is 100 marks.

Question 1 [5 + 5 + 3 = 12 marks]

This question involves first order differential equations (DEs).

(a) **Solve** the DE $y' - 3x/y = 0$.

(b) **Check** that the solution of $y' - 3y/x = t^2$ is $y = x^3\ln(x) + kx^2$.

(c) If a boundary condition of the DE in (b) is $y(1) = -5$, what is the solution of the DE?

Question 2 [8 + 3 + 5 = 12 marks]

This question involves the differential equation (DE) $y'' + 9y = 2x + 1$.

(a) This DE is a "non-homogeneous, second order linear differential equation with constant coefficients". Explain the four parts of this description using the given DE as an example.

(b) There are three stages in solving a DE of this type. What are they?

(c) Obtain the solution of **one** of the first two stages for the given DE.

Question 3 [2 + 2 + 2 + 6 [3+3]= 12 marks]

This question involves basic results of vectors.

(a) If $\underset{\sim}{a} = \langle 1, 8, 4 \rangle$ and $\underset{\sim}{b} = \langle 3, -4, 0 \rangle$, find $2\underset{\sim}{a} - \underset{\sim}{b}$ in component form.

(b) If $\underset{\sim}{c}$ and $\underset{\sim}{d}$ are two 2D position vectors, represent $2\underset{\sim}{c} - \underset{\sim}{d}$ on a diagram.

(c) Find the **unit vector** in the opposite direction to $\langle 1, -8, -4 \rangle$.

(b) A 3D vectors joins coordinate (2,0,5) to (3,1,8). A second vector joins (5,1,3) to (2,1,5).

(i) Are the two vectors orthogonal? Explain.

(ii) Find a third vector that is orthogonal to each of the two vectors.

Question 4 [3 + 3 + 8 [2+3+3] = 14 marks]

This question involves the application of vectors to 3D problems.

(a) Find the equation of the plane that passes through the point (2,5,3) and has normal $\langle 1,5,4 \rangle$.

(b) Provide an argument involving **normal vectors** to demonstrate that the planes represented by the following equations are parallel:

$$3x - 2y + z = 4$$
$$-9x + 6y - 3z = 2$$

(c) An eye is located at the coordinate (−4, 2, 3). The eye views a corner of a cube which is located at (5,4,3). A viewing plane between the eye and the cube is perpendicular to the x-axis and is 3 units from the eye.

(i) Find a vector in the direction of the line joining the eye to the corner of the cube.

(ii) Find a vector equation of the line joining the eye to the corner of the cube.

(iii) Using your answer to (ii) or otherwise, find the (y,z) coordinates of the point on the viewing plane which is on the line joining the eye to the corner of the cube.

Question 5 [3 + 3 + 6 + 2 = 14 marks]

This question involves the basic properties of matrices, and uses the three matrices

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

(a) Find (if possible):

(i) C^T (where T indicates the transpose operation).

(ii) CB.

(iii) $B^T(B+2C)$.

(b) Find the determinant of A.

(c) Find the inverse of A.

(d) How could you **use your answer in (c)** to solve this set of simultaneous equations: [Don't do it, just say how you could do it.]

$$\begin{aligned} x + 4y + 2z &= 4 \\ 2x \qquad\quad\;\; 2z &= 1 \\ x - 2y + \;\; z &= 4 \end{aligned}$$

Question 6 [2 + 2 + 8 = 12 marks]

This question involves the use of augmented matrices.

(a) A set of equations is represented by this matrix: $\left(\begin{array}{ccc|c} 3 & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 4 \end{array}\right)$.

(i) We say the matrix is in echelon form. Explain why it is in echelon form?

(ii) The set of equations represented by this matrix has no solution.
Use the augmented matrix above to explain why.

(c) Use an augmented matrix and elementary row operations to find two different solutions of this set of simultaneous equations:

$$\begin{aligned} 3p + q - 2r &= 5 \\ -2p + q + r &= 2 \\ p + 2q - r &= 7 \end{aligned}$$

Question 7 [3 + 4 + 5 = 12marks]

This question involves the definition, properties, and application of Laplace Transforms (LTs).

(a) Use the LT **formula** to find the LT of $f(t) = -4$.

(b) Use the table of LTs attached to the question paper to find the following (with explanations):

(i) The LT of $4 + 3t^2 - 5\exp(2t)$

(ii) The inverse LT of $5s/(2s^2 - 72)$

(c) Solve the DE $y' + 4y = 4\exp(2t)$ where $y(0) = 0$ using LTs.

Question 8 [3 + 5 + 4 = 12 marks]

This question uses to the non-continuous function $k(t) = \begin{cases} 0 & t<5 \\ t-3 & t\geq 5 \end{cases}$

(a) Sketch the function $k(t)$.

(b) Use either the Laplace Transform (LT) formula or your table of transforms referencing the Heaviside (ie, unit step) function to find the LT of $k(t)$. Call this $K(s)$.

(c) Suppose a function $g(t)$ satisfies $g' - g = 3 + k(t)$ where $g(0) = 1$.
Write down an **expression for G(s)** in terms of $K(s)$. (Do not solve for g.)

----- End of Exam -----

Short Table of Laplace Transforms

f(t)	F(s)
1	$1/s$
e^{at}	$1/(s - G\ a)$
t^n $n = 1, 2, \ldots$	$n!/s^{n+1}$
$\sin(at)$	$a/(s^2 + a^2)$
$\cos(at)$	$s/(s^2 + a^2)$
$\sinh(at)$	$a/(s^2 - a^2)$
$\cosh(at)$	$s/(s^2 - a^2)$
$e^{at} \sin(bt)$	$b/((s - a)^2 + b^2)$
$e^{at} \cos(bt)$	$(s - a)/((s - a)^2 + b^2)$
$t^n e^{at}$ $n = 1, 2, \ldots$	$n!/(s - a)^{n+1}$
$u_c(t)$ (heaviside)	e^{-cs}/s
$f(ct)$	$F(s/c) / c$
$f'(t)$	$s F(s) - f(0)$
$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$
$u_c(t) f(t - c)$	$e^{-cs} F(s)$