THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE
FIRST SEMESTER EXAMINATION - JUNE 2021
SECOND YEAR ENGINEERING
EN212 - ENGINEERING MATHEMATICS III
TIME ALLOWED: 3 HOURS

- **INFORMATION FOR CANDIDATES**

1. Write your name and student number clearly on the front of the examination answer booklets.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains **five(5)** questions. You must attempt **all**. The questions can be done in any order but all parts to the same question must be kept together.
4. Show all working.
5. All answers must be written in examination answer booklet(s) provided. No other written materials will be accepted.
6. Start the answer for each question on a new page. Do not use red ink.
7. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
8. Scientific and business calculators are allowed in the examination room.
9. The last two pages contains information sheet for student use.

- **MARKING SCHEME**

Marks are indicated in brackets for each question. Total is **73 marks** with 50% weight.

Question 1 [9+4=13 marks]

(a) Find $\mathcal{F}_c\{e^{-x}\sin(x)\}$,

(b) Use

$$\mathcal{F}_c\{f''(x)\} = -\frac{2}{\sqrt{2\pi}}f'(0) - w^2\mathcal{F}_c\{f(x)\}$$

and also part (a) to find $\mathcal{F}_c\{e^{-x}\cos(x)\}$.

Question 2 [10 marks]

The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

Question 3 [11+4=15 marks]

(a) Find the Fourier series of the function

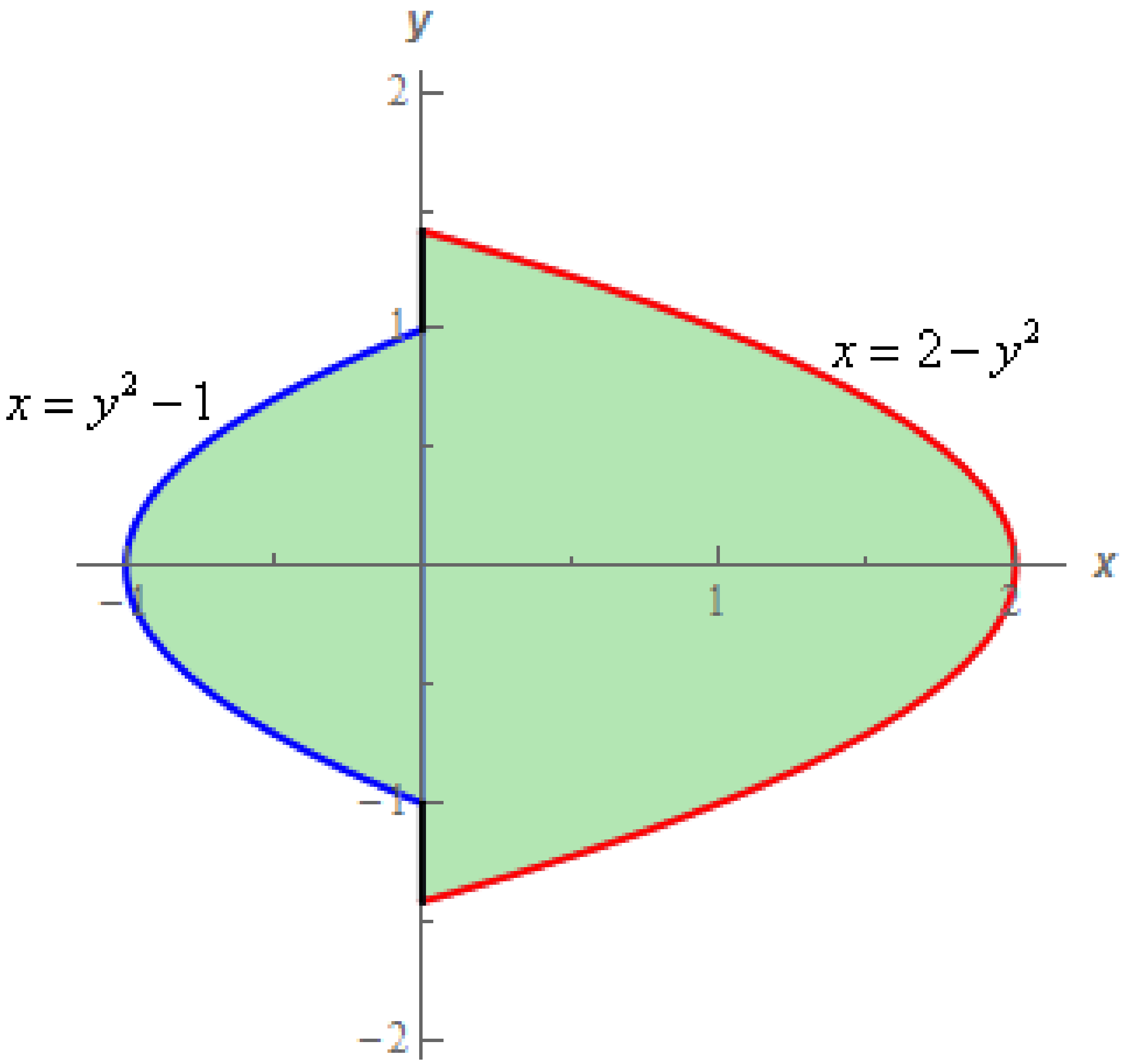
$$f(x) = \begin{cases} \pi & -\pi \leq x < 0 \\ 2x & 0 < x \leq \pi \end{cases},$$

(b) Use part (a) to find the value of

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81} + \cdots$$

Question 4 [20 marks]

Evaluate $\iint_D 6y^2 + 10yx^4\, dA$ where D is is the region shown below:



Question 5 [10+5=15 marks]

(a) Determine that $\vec{F} = x^2y\,\vec{i} + xyz\,\vec{j} - x^2y^2\,\vec{k}$ is a conservative vector field,

(b) Find the flux of the vector field $F(x, y, z) = y\vec{i} - x\vec{j} + z\vec{k}$ through the surface $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 2$, oriented upwards.

End of Exam

Reference Material

(1)

$$\mathcal{F}_c\{f(x)\} = \hat{f}_c(w) = \tfrac{2}{\sqrt{2\pi}} \int_0^\infty f(x)\cos(wx)dx, \qquad w > 0,$$

(2)

$$\sin(x)\cos(x) = \frac{\sin(2x)}{2}$$

$$\sin(A)\cos(B) = \frac{\sin(A+B)+\sin(A-B)}{2}$$

(3) TRAPEZOIDAL RULE

$$T_n = \frac{\Delta x}{2}\Big[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\Big].$$

(4) MIDPOINT RULE

$$M_n = \sum_{i=1}^{n} f(m_i)\Delta x.$$

(5) SIMPSON RULE

$$S_n = \frac{\Delta x}{3}\Big[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots +2f(x_{n-2})+4f(x_{n-1})+f(x_n)\Big].$$

(6) Fourier series for a function $f(x)$ with period 2π is defined as

$$f(x) = a_0 + \sum_{n=1}^{\infty}\Big(a_n\cos(nx)+b_n\sin(nx)\Big),$$

such that coefficients a_n and b_n can be find by using following formulas:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx, \qquad n = 1,2,3,\cdots,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx, \qquad n = 1,2,3,\cdots.$$

(7) Let $f(x)$ be periodic with period 2π and piece-wise continuous in $[\pi,\pi]$, and have a left-hand derivative and a right-hand derivative at each point of that interval. Then the Fourier series of $f(x)$ is convergent and

$$\textbf{The sum of the series} = \begin{cases} f(x_0) & f \text{ is continuous at } x_0 \\ \frac{1}{2}\Big[f(x_0+0)+f(x_0-0)\Big] & f \text{ is discontinuous at } x_0 \end{cases}$$

(8)

- If the surface S is oriented outward, then

$$\begin{aligned}\iint_S \mathbf{F}(x,y,z)\cdot d\mathbf{S} &= \iint_S \mathbf{F}(x,y,z)\cdot \mathbf{n}dS \\ &= \iint_{D(u,v)} \mathbf{F}(x(u,v),y(u,v),z(u,v))\cdot\left[\frac{\partial \mathbf{r}}{\partial u}\times\frac{\partial \mathbf{r}}{\partial v}\right]dudv;\end{aligned}$$

- If the surface S is oriented inward, then

$$\begin{aligned}\iint_S \mathbf{F}(x,y,z)\cdot d\mathbf{S} &= \iint_S \mathbf{F}(x,y,z)\cdot \mathbf{n}dS \\ &= \iint_{D(u,v)} \mathbf{F}(x(u,v),y(u,v),z(u,v))\cdot\left[\frac{\partial \mathbf{r}}{\partial v}\times\frac{\partial \mathbf{r}}{\partial u}\right]dudv;\end{aligned}$$

(9)

$$curl\vec{F} = \nabla\times\vec{f} = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R\end{vmatrix}$$

where $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j} + R(x,y)\vec{k}$