THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATION - JUNE 2022

SECOND YEAR ENGINEERING

EN212 - ENGINEERING MATHEMATICS III

TIME ALLOWED: 3 HOURS

- **INFORMATION FOR CANDIDATES**

1. Write your name and student number clearly on the front of the examination answer booklets.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains **four(4)** questions. You must attempt **all**. The questions can be done in any order but all parts to the same question must be kept together.
4. Show all working.
5. All answers must be written in examination answer booklet(s) provided. No other written materials will be accepted.
6. Start the answer for each question on a new page. Do not use red ink.
7. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
8. Scientific and business calculators are allowed in the examination room.
9. The last two pages contains information sheet for student use.

- **MARKING SCHEME**

Marks are indicated in brackets for each question. Total is **65 marks** with 50% weight.

Question 1 [6+10=16 marks]

(a) Use Greens Theorem to evaluate $\oint_C xydx + x^2y^3dy$ where C is the triangle with vertices $(0,0), (1,0)$ and $(1,2)$ with positive orientation,

(b) Find the flux of the vector field $F(x,y,z) = y\vec{i} + x\vec{j} + z\vec{k}$ through the surface S, parameterized by the vector $\vec{r}(u,v) = \cos(v)\vec{i} + \sin(v)\vec{j} + u\vec{k}$ with $0 \leq u \leq 2$ and $\frac{\pi}{2} \leq v \leq \pi$.

Question 2 [5+9+6=20 marks]

(a) Use Fourier integral representation of the function

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

to find $\displaystyle\int_0^\infty \frac{\sin(x)}{x} dx$,

(b) Using Fourier series of $f(x) = x^2$ on $-\pi < x < \pi$ to find A and B such that

$$\begin{aligned} A &= 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \\ B &= 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \end{aligned}$$

(c) Use Fourier transform to solve the initial-boundary value problem

$$\begin{aligned} u_t &= u_{xx} & -\infty < x < \infty, \quad t > 0, \\ u(x,0) &= \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}. \end{aligned}$$

Question 3 [8+5=13 marks]

(a) Solve the integral $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 \sin(\pi x^3)\, dx\, dy$,

(b) If $f(x,y) = x^2 \sin\left(\frac{y}{x}\right) + y^2 \cos\left(\frac{y}{x}\right)$ then find $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

Question 4 [5+(5+3+3)=16 marks]

(a) The sum of two numbers is 20. If each number is added to its square, the product of the two sums is 155.55. Use Newton-Raphson method with 2 iterations, starting point at 19 to find these numbers,

(b) Approximate the $\int_1^{1.5} x^2 \ln(x)dx$ using the

i. M_4,

ii. T_4,

iii. S_4.

End of Exam

Reference Material

(1) **GREEN'S THEOREM**

Let C be a positively oriented, piece-wise smooth, simple, closed curve and let D be the region enclosed by the curve. If P and Q have continuous first order partial derivatives on D then,

$$\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

(2) The surface integral of the vector field F over the oriented surface S (or the flux of the vector field F across the surface S) can be written in one of the following forms:

- the surface S is oriented outward, then

$$\begin{aligned}\iint_S \mathbf{F}(x,y,z)\cdot d\mathbf{S} &= \iint_S \mathbf{F}(x,y,z)\cdot \mathbf{n} dS \\ &= \iint_{D(u,v)} \mathbf{F}(x(u,v), y(u,v), z(u,v))\cdot\left[\tfrac{\partial \mathbf{r}}{\partial u}\times\tfrac{\partial \mathbf{r}}{\partial v}\right] dudv;\end{aligned}$$

- the surface S is oriented inward, then

$$\begin{aligned}\iint_S \mathbf{F}(x,y,z)\cdot d\mathbf{S} &= \iint_S \mathbf{F}(x,y,z)\cdot \mathbf{n} dS \\ &= \iint_{D(u,v)} \mathbf{F}(x(u,v), y(u,v), z(u,v))\cdot\left[\tfrac{\partial \mathbf{r}}{\partial v}\times\tfrac{\partial \mathbf{r}}{\partial u}\right] dudv;\end{aligned}$$

(3) **TRAPEZOIDAL RULE**

$$T_n = \frac{\Delta x}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\Big].$$

(4) **MIDPOINT RULE**

$$M_n = \sum_{i=1}^{n} f(m_i)\Delta x.$$

(5) **SIMPSON RULE**

$$\begin{aligned}S_n =& \frac{\Delta x}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots \\ &+ 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big].\end{aligned}$$

(6) Fourier series for a function $f(x)$ with period 2π is defined as

$$f(x) = a_0 + \sum_{n=1}^{\infty}\Big(a_n\cos(nx) + b_n\sin(nx)\Big),$$

such that coefficients a_n and b_n can be find by using following formulas:

$$
\begin{aligned}
a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx, \\
a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx, \qquad n = 1, 2, 3, \cdots, \\
b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx, \qquad n = 1, 2, 3, \cdots.
\end{aligned}
$$

(7) Let $f(x)$ be periodic with period 2π and piece-wise continuous in $[\pi, \pi]$, and have a left-hand derivative and a right-hand derivative at each point of that interval. Then the Fourier series of $f(x)$ is convergent and

$$
\textbf{The sum of the series} = \begin{cases} f(x_0) & f \text{ is continuous at } x_0 \\ \frac{1}{2}\Big[f(x_0+0) + f(x_0-0)\Big] & f \text{ is discontinuous at } x_0 \end{cases}
$$

(8) Fourier integral for the function $f(x)$ will be define as

$$
f(x) = \int_0^{\infty} \Big[A(s)\cos(sx) + B(s)\sin(sx)\Big] ds
$$

such than

$$
\begin{aligned}
A(s) &= \tfrac{1}{\pi}\int_{-\infty}^{\infty} \Big[f(x)\cos(sx)\Big] dx \\
B(s) &= \tfrac{1}{\pi}\int_{-\infty}^{\infty} \Big[f(x)\sin(sx)\Big] dx.
\end{aligned}
$$

Moreover if

$$
\begin{aligned}
f(-x) = +f(x) &\implies B(s) = 0 \qquad \text{even function} \\
f(-x) = -f(x) &\implies A(s) = 0 \qquad \text{odd function}
\end{aligned}
$$

(9)

$$
\begin{aligned}
\mathcal{F}\{f(x)\} &= \hat{f}(w) = \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx, , \\
\mathcal{F}\{g(x)\} &= \hat{g}(w) = \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) e^{-iwx} dx, , \\
\mathcal{F}\{u(x,t)\} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,t) e^{-iwx} dx, \\
\mathcal{F}\{u(x,t)\} &= \hat{u}(w,t) \longrightarrow \mathcal{F}\{u(x,a)\} = \hat{u}(w,a), \\
\mathcal{F}\{u_t(x,t)\} &= \frac{\partial \hat{u}(w,t)}{\partial t} \longrightarrow \mathcal{F}\{u_t(x,a)\} = \hat{u}_t(w,a), \\
\mathcal{F}\{u_{tt}(x,t)\} &= \frac{\partial^2 \hat{u}(w,t)}{\partial t^2}, \\
\mathcal{F}\{u_x(x,t)\} &= iw\ \hat{u}(w,t), \\
\mathcal{F}\{u_{xx}(x,t)\} &= -w^2\ \hat{u}(w,t), \\
\mathcal{F}^{-1}\{\hat{u}(w,t)\} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{u}(w,t) e^{iwx} dw, \\
u(x,t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{u}(w,t) e^{iwx} dw.
\end{aligned}
$$

(10) Solution of differential equation $\frac{\partial y}{\partial t} + p(t)y = q(t)$ where $p(t)$ and $q(t)$ are continuous functions, is

$$y(t) = \frac{\int \mu(t)q(t)dt + c}{\mu(t)},$$

such that $\mu(t) = e^{\int p(t)dt}$.

(11) NEWTON METHOD

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$