THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

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DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATION - MAY 2023

SECOND YEAR BACHELOR OF ENGINEERING

EN212 - ENGINEERING MATHEMATICS III

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklets.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains **six(6)** questions. You must attempt **all**. The questions can be done in any order but all parts to the same question must be kept together.
4. Show all working.
5. All answers must be written in examination answer booklet(s) provided. No other written materials will be accepted.
6. Start the answer for each question on a new page. Do not use red ink.
7. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
8. Scientific and business calculators are allowed in the examination room.
9. The last two pages contains information sheet for student use.

MARKING SCHEME

Marks are indicated in brackets for each question. Total is **78 marks** with 50% weight.

Question 1 [8 marks]

Trapezoidal rule T_2 gives $\int_0^1 f(x)dx = 2$ and also using Trapezoidal rule T_4 gives $\int_0^1 f(x)dx = 1.75$. If $f(0.25) = f(0.75) = \alpha$ then what is the value of α?

Question 2 [5 + 5 + 4 + 1=15 marks]

Find an approximation to $\sqrt[3]{7}$ with **two iterations** using

(a) Bisection method on $[1, 2]$,

(b) Newton-Raphson method with starting point at $x_0 = 1.5$

(c) Secant method with starting points at $x_0 = 1$ and $x_1 = 2$,

(d) Considering the exact value for $\sqrt[3]{7} \approx 1.91293$, which method is the best?

Question 3 [10 marks]

Find the Fourier integral formula for

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ x & 0 < x < 1 \\ 0 & 1 < x < \infty \end{cases},$$

to evaluate $\displaystyle\int_0^\infty \frac{1-\cos(w)}{w^2} dw$.

Question 4 [20 + 5=25 marks]

(a) Evaluate $\int\int_R xy \, dA$ where R is a region obtained by points $A(1,1), B(2,2), C(3,1)$ and $D(2,0)$.

(b) If $f(x,y) = x^2 \sin\left(\frac{y}{x}\right) + y^2 \cos\left(\frac{y}{x}\right)$ then find $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

Question 5 [10 marks]

Use Fourier series of the function $f(x) = \begin{cases} 1 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \text{else} \end{cases}$ to find the value of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots.$$

Question 6 [10 marks]

Find Fourier sine transform of $f(x) = e^{-|x|}$ then using that evaluate

$$\int_0^{\infty} \frac{x \sin(\pi x)}{1 + x^2} dx.$$

End of Exam

Reference Material

(1) TRAPEZOIDAL RULE

$$T_n = \frac{\Delta x}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\Big].$$

(2) Fourier series for a function $f(x)$ with period 2π is defined as

$$f(x) = a_0 + \sum_{n=1}^{\infty}\Big(a_n\cos(nx) + b_n\sin(nx)\Big),$$

such that coefficients a_n and b_n can be find by using following formulas:

$$\begin{aligned} a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx, \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx, \qquad n = 1, 2, 3, \cdots, \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx, \qquad n = 1, 2, 3, \cdots. \end{aligned}$$

(3) Let $f(x)$ be periodic with period 2π and piece-wise continuous in $[\pi, \pi]$, and have a left-hand derivative and a right-hand derivative at each point of that interval. Then the Fourier series of $f(x)$ is convergent and

$$\textbf{The sum of the series} = \begin{cases} f(x_0) & f \text{ is continuous at } x_0 \\ \frac{1}{2}\Big[f(x_0+0) + f(x_0-0)\Big] & f \text{ is discontinuous at } x_0 \end{cases}$$

(4) Fourier integral for the function $f(x)$ will be define as

$$f(x) = \int_0^{\infty}\Big[A(s)\cos(sx) + B(s)\sin(sx)\Big]ds$$

such than

$$\begin{aligned} A(s) &= \tfrac{1}{\pi}\int_{-\infty}^{\infty}\Big[f(x)\cos(sx)\Big]dx \\ B(s) &= \tfrac{1}{\pi}\int_{-\infty}^{\infty}\Big[f(x)\sin(sx)\Big]dx. \end{aligned}$$

Moreover if

$$\begin{aligned} f(-x) = +f(x) &\implies B(s) = 0 \quad \text{even function} \\ f(-x) = -f(x) &\implies A(s) = 0 \quad \text{odd function} \end{aligned}$$

(5) NEWTON METHOD

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

(6) SECANT METHOD

$$x_{i+1} = x_i - \frac{f(x_i)}{Q(x_{i-1}, x_i)}$$

where

$$Q(x_{i-1}, x_i) = \frac{f(x_{i-1}) - f(x_i)}{x_{i-1} - x_i}.$$

(7) Equation of line passing the points $A(x_0, y_0)$ and $B(x_1, y_1)$ is

$$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0}\left(x - x_0\right).$$

(8) The Fourier sine transform for the function $f(x)$ will be denoted by $\mathcal{F}_s\{f(x)\}$ or $\hat{f}_s(w)$ and it is equal to

$$\mathcal{F}_s\{f(x)\} = \hat{f}_s(w) = \frac{2}{\sqrt{2\pi}} \int_0^\infty f(x)\sin(wx)dx, \qquad w > 0,$$

and inverse Fourier sine transform for the function $\hat{f}_s(w)$ is equal to

$$f(x) = \mathcal{F}_s^{-1}\{\hat{f}_s(w)\} = \frac{2}{\sqrt{2\pi}} \int_0^\infty \hat{f}_s(w)\sin(wx)dw, \quad x > 0.$$