THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS – 2023

FIRST YEAR BACHELOR IN APPLIED PHYICS

EN112C – ENGINEERING MATHEMATICS I

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIVE (5) questions. You are to **answer ALL** the questions.
4. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.
8. The last page contain formula sheet for student information.

MARKING SCHEME

Marks are indicated at the beginning of each question. The total is **100 marks with 50% weightage.**

QUESTION 1 [5 + 4 + 3 + 8 = 20 marks]

(a) If $f(x)=\sqrt{x}\ ;x\geq 0$, then determine $f'(4)$ from the definition of derivative.

(b) Show that the function $f(x)=\begin{cases}1-3x & ;x<1\\ 2-4x & ;x\geq 1\end{cases}$

is not differentiable at $x=1$.

(c) Determine $\frac{dy}{dx}$ if $y = e^{3sinx-2tanx}$

(d) A water tank has the shape of an inverted circular cone with base radius $2m$ and height $4m$. If the water is being pumped into the tank at a rate of $2m^3/\text{min}$, find the rate at which the water level is rising up when the water is 3m deep.

QUESTION 2 [5 + 5 + 4 + 6 = 20 marks]

(a) By using the definition of definite integral evaluate $\int_0^1 (x^3-3x^2+5)dx$

(b) Use the midpoint rule with $n=5$, to approximate $\int_0^2 \frac{dx}{\sqrt{x}}$

(c) Integrate $\int x^2 \ln x\, dx$

(d) Determine the volume of a right circular cone of radius 'r' and slant height $\sqrt{h^2+r^2}$, by using the method of surface of revolution.

QUESTION 3 [4 + 6 + 5 + 5 = 20 marks]

(a) If $z=-1+3i$, then determine $Re\left(\frac{z\bar{z}}{|z|^2}\right)$.

(b) Write the complex number $z=-\sqrt{3}+3i$ in polar form.

(c) Simplify: $(1+i)^8$

(d) Solve: $z^3=\sqrt{3}-\sqrt{3}i$, where z is a complex number and $i=\sqrt{-1}$.

QUESTION 4 [5 + 5 + 5 + 5 = 20 marks]

(a) Determine the median class of the following frequency distribution.

Class interval	Frequency
70-85	6
86-101	8
102-117	4
118-133	6
134-149	10

(b) 2.01% of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 200 screws chosen at random exactly 3 will be defective.

(c) As per the survey done on Covid infection in each family in a certain region the data has been arranged in the following table. Calculate χ^2 value.

Number of infected person	O_i	E_i
One	30	28
Two	25	27
Three	20	22
Four	18	14
Five	08	05
Six	04	02

(d) Determine the line of regression from the following data and hence estimate y corresponding

to $x = 0.2567$

x	1	3	8	9	12
y	5	4	4	2	1

QUESTION 5 [6 + 4 + 5 + (3 + 2) = 20 marks]

(a) Find the local extrema of $f(x) = x^3 + 3x^2 + 6$ by using the first derivative test.

(b) Evaluate: $\int_0^7 |x - 3|\, dx$

(c) Express the complex number $-\sqrt{3} - \sqrt{3}\, i$ in exponential form.

(d) Tests on inductors indicates that breakdown voltages were normally distributed about a mean of 160kv and standard deviation of 40kv. Determine the probability of inductance whose breakdown voltage (given: $P(0 \le z \le 0.5) = 0.1915$)

(i) Exceeds 140kv

(ii) Lies between 140kv and 180kv

END OF EXAM

FORMULAE

$\frac{d}{dx}\{C\}=0$; Where C is a constant.	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}\{x^n\}=nx^{n-1}$	$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$
$\frac{d}{dx}\{e^x\}=e^x$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$
$\frac{d}{dx}\{a^x\}=a^x \ln a$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$
$\frac{d}{dx}\{\sin x\}=\cos x$	$\int \operatorname{cosec} x\,dx = \ln\lvert\operatorname{cosec} x - \cot x\rvert + c$
$\frac{d}{dx}\{\cos x\}=-\sin x$	$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + c$
$\frac{d}{dx}\{\tan x\}=\sec^2 x$	$\int \frac{-1}{\sqrt{1-x^2}}dx = \cos^{-1} x + c$
$\frac{d}{dx}\{\cot x\}=-\operatorname{cosec}^2 x$	$\int \frac{1}{1+x^2}dx = \tan^{-1} x + c$
$\frac{d}{dx}\{\sec x\}=\sec x.\tan x$	$\int \frac{-1}{1+x^2}dx = \cot^{-1} x + c$
$\frac{d}{dx}\{\operatorname{cosec} x\}=-\operatorname{cosec} x.\cot x$	$\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1} x + c$
$\frac{d}{dx}\{\ln x\}=\frac{1}{x}$	$\int \frac{-1}{x\sqrt{x^2-1}}dx = \operatorname{cosec}^{-1} x + c$
$\frac{d}{dx}\{\tan^{-1} x\}=\frac{1}{1+x^2}$	$\int_a^b f(x)dx = F(b)-F(a)$; Where $F'(x)=f(x)$
$\frac{d}{dx}\{\sin^{-1} x\}=\frac{1}{\sqrt{1-x^2}}$	$\int_a^b f(x)dx = \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f\left(a+\frac{i(b-a)}{n}\right)$
$\int x^n dx = \frac{x^{n+1}}{n+1}+c;\quad n\neq -1$	$\int_a^b f(x)dx = \sum_{i=1}^{n} f(\overline{x_i})\Delta x;\ \Delta x = \frac{b-a}{n}, \overline{x_i} = \frac{x_{i-1}+x_i}{2}$
$\int e^x dx = e^x + c$	$\int (u.v)dx = u\int v dx - \int\left\{\frac{d(u)}{dx}\int v dx\right\}dx + c$
$\int a^x dx = \frac{a^x}{\ln a}+c$	$\sin^2 x + \cos^2 x = \sec^2 x - \tan^2 x = \operatorname{cosec}^2 x - \cot^2 x = 1$
$\int \frac{1}{x}dx = \ln x + c$	$e^{i\theta} = \cos\theta + i\sin\theta$
$\int \sin x\,dx = -\cos x + c$	Normal equations of $y=a+bx$ are $\sum y = na + b\sum x,\quad \sum xy = a\sum x + b\sum x^2$