



THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS – 2022
FIST YEAR BACHELOR IN APPLIED SCIENCE

MA115 – MATHEMATICS 1 AS (A)

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES

1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIVE (5) questions. You are to **answer ALL** the questions.
4. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.
8. A formula sheet is attached.

MARKING SCHEME

Marks are indicated at the beginning of each question. The total is **100 marks**.

**DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
MA115 FORMULA SHEET**

STATISTICS AND PROBABILITY

Statistics and Probability	
<p><u>Grouped data – Center of data</u></p> <ol style="list-style-type: none"> 1. $Me = LL + W(N/2 - CB)/F$ 2. $Mo = LL + Wd_1/(d_1 + d_2)$ 	<p><u>Grouped data – Spread of data</u></p> <ol style="list-style-type: none"> 3. $Q_1 = LL + W(N/4 - CB)/F$ 4. $Q_3 = LL + W(3N/4 - CB)/F$ 5. $Variance = \frac{(\sum FX^2)}{N} - \left(\frac{\sum FX}{N}\right)^2$

Standard Integrals	Standard derivatives
<ol style="list-style-type: none"> 1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad n \neq 1$ 2. $\int \cos x dx = \sin x + c$ 3. $\int \sin x dx = -\cos x + c$ 4. $\int \sec^2 x dx = \tan x + c$ 5. $\int \frac{1}{x} dx = \ln x + c \quad x \neq 0$ 6. $\int e^x = e^x + c$ 7. $e^{ax+b} = \frac{1}{a} e^{ax+b}$ 	<ol style="list-style-type: none"> 1. $\frac{d}{dx}(x^n) = nx^{n-1} \quad n \neq 1$ 2. $\frac{d}{dx}(\sin x) = \cos x$ 3. $\frac{d}{dx}(\cos x) = -\sin x$ 4. $\frac{d}{dx}(\tan x) = \sec^2 x$ 5. $\frac{d}{dx}(e^x) = e^x$ 6. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
$\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos(2\theta)$	$\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos(2\theta)$

Basic Rules of Differentiation

First Derivative	$f'(x) = \lim_{h \rightarrow \infty} \frac{f(x+h) - f(x)}{h}$
Product Rule	$(f(x) \times g(x)) = f(x) \times g'(x) + g(x) \times f'(x)$
Chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Quotient Rule	$\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \times f'(x) - f(x) \times g'(x)}{[g(x)]^2}$

QUESTION 1 [(3 + 4 + 9) + 8 = 24 marks]

- (a) The winning margin in 100 basketball games were recorded. The results are given in the table below:

Margin (points)	Frequency
1-10	13
11-20	35
21-30	27
31-40	18
41-50	7
TOTAL	100

- (i) Construct a column in your answer booklet and write the cumulative frequency.
(ii) Estimate the median.
(iii) Find the quartile deviation.
- (b) A set of 10 integers $\{3, 9, 5, 5, 6, 4, a, 6, b, 8\}$ has a mean of 6 and a variance of 3.2. Find a and b given that $a > b$.

QUESTION 2 [5 + 10 = 15 marks]

- (a) Find k if the tangent to $y = 2x^2 + kx^2 - 3$ at a point where $x = 2$ and has gradient 4.
(b) Find the equation of the tangent to: $y = \sec x$ at $x = \frac{\pi}{4}$

QUESTION 3 [(4 + 5) + (4 + 3 + 3) = 19 marks]

- (a) Given $f(x) = \sqrt{6-x}$ and $g(x) = 5x - 7$, find:

- (i) $(g \circ g)(x)$
(ii) $(g \circ f)(6)$

- (b) Given the function $f(x) = \frac{1}{\sqrt{x-4}}$.

- (i) Sketch the function.
(ii) Find the domain and range of the function in set notation.

QUESTION 4 [4 + 7 + 7 = 18 marks]

(a) Find $\frac{dy}{dx}$ of $y = 2\left(x^2 - \frac{2}{x}\right)^3$.

(b) Use implicit differentiation to find the gradient of the tangent to: $x + y = 4xy$ at $x = \frac{1}{2}$.

(c) Water is draining from a swimming pool. The remaining volume of water after t minutes is $V = 200(50 - t)^2$ meters square. Find the average rate at which the water leaves the pool in the first 5 minutes.

QUESTION 5 [7 + 5 + (5 + 7) = 24 marks]

(a) Find $\int (2 - \cos x)^2 dx$.

(b) Use substitution to integrate: $(\sqrt{x^3 + x})(3x^2 + 1)$.

(c) Give the functions $y = x^2 - 2x$ and $y = 3$.

(i) Sketch the graphs on the same set of axes.

(ii) Find the area of the region enclosed by the two graphs.