

THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY  
DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

**FIRST SEMESTER EXAMINATIONS – 2023**  
**FIST YEAR BACHELOR IN APPLIED SCIENCE**

**MA115 -- MATHEMATICS 1 AS (A)**

**TIME ALLOWED: 3 HOURS**

**INFORMATION FOR CANDIDATES**

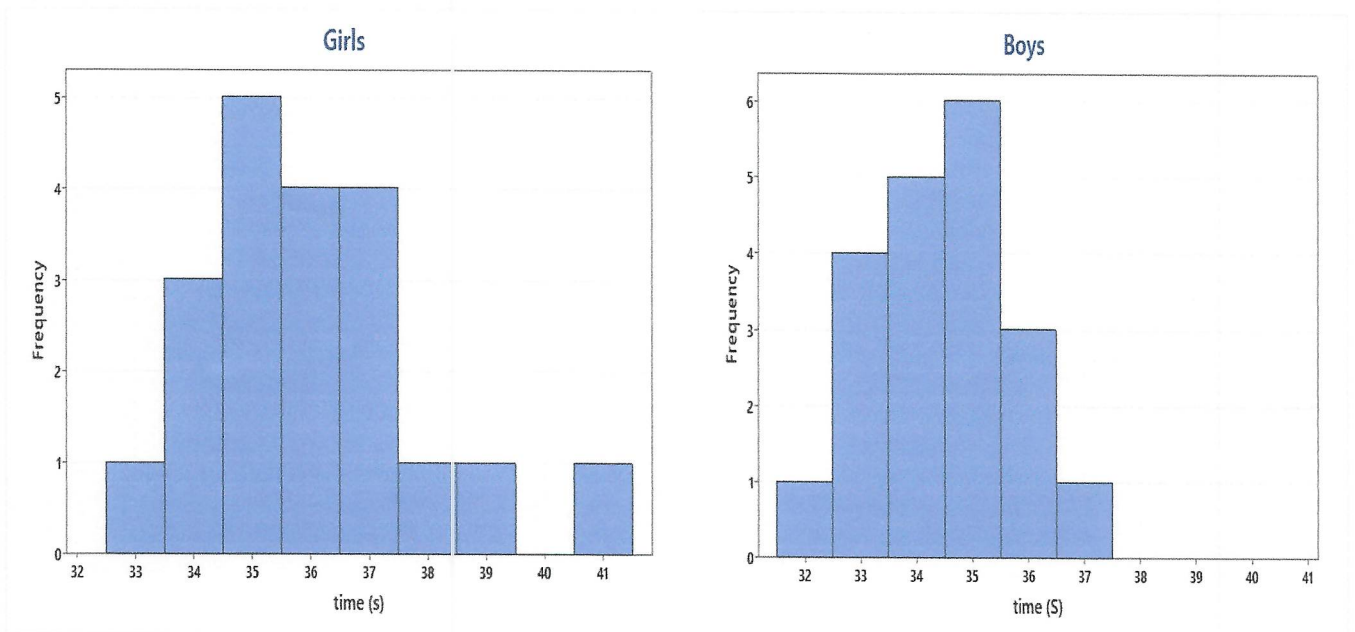
1. Write your name and student number clearly on the front of the examination answer booklet.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains FIVE (5) questions. You are to **answer ALL** the questions.
4. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
5. Start the answer for each question on a **new** page. Do **not** use red ink.
6. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
7. Scientific and business calculators are allowed in the examination room.
8. A formula sheet is attached.

**MARKING SCHEME**

Marks are indicated at the beginning of each question. The total is **100 marks**.

**QUESTION 1** [4 + 6 + 10 + 2 = 22 marks]

The histogram below shows the times for the 100 meter freestyle recorded by members of a swimming squared for two groups.



Using the two histograms answer the following questions:

- (a) Comment on the distribution of both groups.
- (b) Calculate the mean for both groups.
- (c) Calculate the standard deviation for both groups.
- (d) Which group is more consistent?

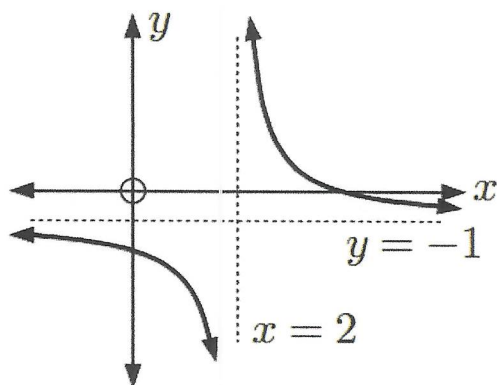
**QUESTION 2** [(4 + 4) + (6 + 6) = 20 marks]

Given  $f(x) = \sqrt{6-x}$  and  $g(x) = 5x - 7$ , find:

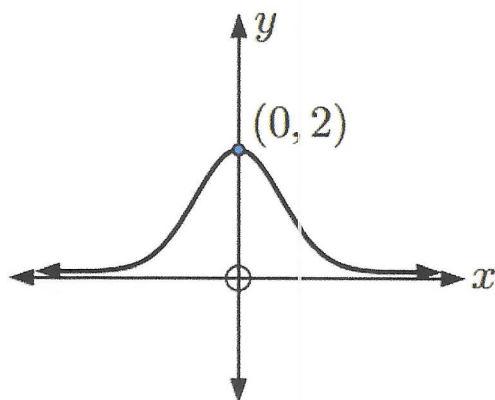
- (i)  $(gog)(x)$
- (ii)  $(gof)(5)$

(b) Find the **domain** and **range** for the following graphs.

(i)



(ii)



**QUESTION 3** [(5 + 7) + 10 = 22 marks]

(a) Find the gradient function of the following functions:

(i)  $y = \frac{1}{\sqrt{2e^{-x} + 2}}$

(ii)  $y = \frac{2\sqrt{x}}{\ln x}$

(b) Find the gradient of the tangent to:  $f(x) = \sin^3 x$  at the point where  $x = \frac{2\pi}{3}$

**QUESTION 4** [8 + 8 + 7 = 23 marks]

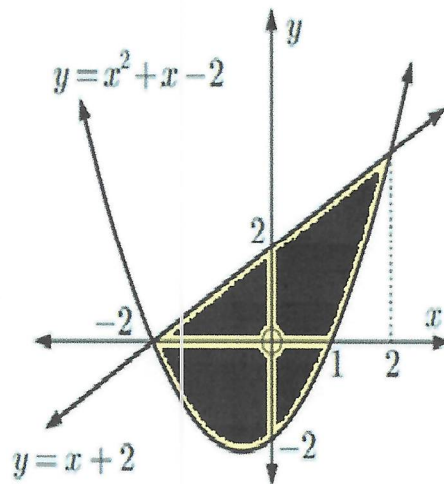
(a) Find the gradient of the tangent to:  $x + y = 8xy$  at  $x = \frac{1}{2}$ .

(b) Find the points of contact where horizontal tangents meet the curve  $y = 2\sqrt{x} + \frac{1}{\sqrt{x}}$ .

(c) Find the equation of the normal to  $y = \frac{1}{(x^2 + 1)^2}$  at  $\left(1, \frac{1}{4}\right)$ .

**QUESTION 5** [13 marks]

Find the shaded area enclosed by the two functions given below:



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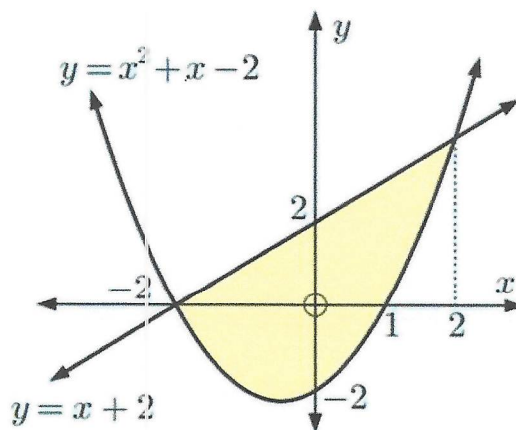
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**DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE  
MA115 FORMULA SHEET**

**STATISTICS AND PROBABILITY**

<b>Statistics and Probability</b>	
<p><u>Grouped data – Center of data</u></p> <ol style="list-style-type: none"> <li>1. <math>Me = LL + W(N/2 - CB)/F</math></li> <li>2. <math>Mo = LL + Wd_1/(d_1 + d_2)</math></li> </ol>	<p><u>Grouped data – Spread of data</u></p> <ol style="list-style-type: none"> <li>3. <math>Q_1 = LL + W(N/4 - CB)/F</math></li> <li>4. <math>Q_3 = LL + W(3N/4 - CB)/F</math></li> <li>5. <math>Variance = \frac{(\sum FX^2)}{N} - \left(\frac{\sum FX}{N}\right)^2</math></li> </ol>

<b>Standard Integrals</b>	<b>Standard derivatives</b>
<ol style="list-style-type: none"> <li>1. <math>\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad n \neq 1</math></li> <li>2. <math>\int \cos x dx = \sin x + c</math></li> <li>3. <math>\int \sin x dx = -\cos x + c</math></li> <li>4. <math>\int \sec^2 x dx = \tan x + c</math></li> <li>5. <math>\int \frac{1}{x} dx = \ln x  + c \quad x \neq 0</math></li> <li>6. <math>\int e^x = e^x + c</math></li> <li>7. <math>e^{ax+b} = \frac{1}{a} e^{ax+b}</math></li> </ol>	<ol style="list-style-type: none"> <li>1. <math>\frac{d}{dx}(x^n) = nx^{n-1} \quad n \neq 1</math></li> <li>2. <math>\frac{d}{dx}(\sin x) = \cos x</math></li> <li>3. <math>\frac{d}{dx}(\cos x) = -\sin x</math></li> <li>4. <math>\frac{d}{dx}(\tan x) = \sec^2 x</math></li> <li>5. <math>\frac{d}{dx}(e^x) = e^x</math></li> <li>6. <math>\frac{d}{dx}(\ln x) = \frac{1}{x}</math></li> </ol>
$\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos(2\theta)$	$\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos(2\theta)$

**Basic Rules of Differentiation**

First Derivative	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
Product Rule	$(f(x) \times g(x))' = f(x) \times g'(x) + g(x) \times f'(x)$
Chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Quotient Rule	$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) \times f'(x) - f(x) \times g'(x)}{[g(x)]^2}$