THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS – 2022

FIRST YEAR BACHELOR OF SCIENCE IN APPLIED CHEMISTRY

FIRST YEAR BACHELOR OF SCIENCE IN FOOD TECHNOLOGY

MA125 – MATHEMATICS 1B (AS)

TIME ALLOWED: 3 HOURS

INFORMATION FOR CANDIDATES:

1. Write your name and student number clearly on the front of the examination answer booklet/s.
2. You have 10 minutes to read this paper. You must not begin writing during this time.
3. This paper contains five (5) questions. You should attempt all the questions.
4. Make sure you have 4 pages, including cover page and formula sheet.
5. All answers must be written in examination answer booklets provided. No other written materials will be accepted.
6. Start the answer for each question on a new page.
7. Do not use red ink or pencil.
8. Notes, textbooks, mobile phones and other recording devices are not allowed in the examination room.
9. Scientific and business calculators are allowed in the examination room.

MARKING SCHEME

Marks are indicated at the beginning of each question. Total mark is 100.

FORMULA SHEET/INFORMATION GUIDE

1 $\log_a y = x$ or $y = a^x$

2. $y = a^{kx} + b$

(i) Graph of y is an exponential growth curve if a>1 and k>0.

(ii) Graph of y is an exponential decay curve if a>1 and k<0.

(iii) Asymptote: y = b

(3) Half life: $T = -\dfrac{\ln 2}{k}$ Doubling time: $T = \dfrac{\ln 2}{k}$

(4) $y = y_0 e^{kx}$ (5) $y = f(x)\cdot g(x),\ \dfrac{dy}{dx} = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

(6) $y = \dfrac{f(x)}{g(x)},\quad \dfrac{dy}{dx} = \dfrac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$

(7) $y = f(u)$, where u is a function of x. Then $\dfrac{dy}{dx} = f'(u)\cdot\dfrac{du}{dx}$ (8) $y = e^{ax},\ \dfrac{dy}{dx} = ae^{ax}$

(9) $y = x^n,\ \dfrac{dy}{dx} = nx^{n+1} + C, n \neq -1$ (10) $\int f(x)dx = F(x) + C$ (11) $\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$

(12) $\int kf(x)dx = k\int f(x)dx$ where k is a constant. (13) $\int x^n dx = \dfrac{1}{n}x^{n+1} + C, n \neq -1$

(14) $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$ (15) $\int (ax+b)^n dx = \dfrac{1}{a(n+1)}(ax+b)^{n+1} + C, n \neq -1$

(16) $\int \dfrac{1}{ax+b}dx = \dfrac{1}{a}\ln|ax+b| + C$ (17) $\int e^{ax}dx = \dfrac{1}{a}e^{ax} + C$ (18) $\int \cos(ax+b)dx = \dfrac{1}{a}\sin(ax+b) + C$

(19) $\int \sin(ax+b)dx = -\dfrac{1}{a}\cos(ax+b) + C$ (20) $\int \tan(ax+b)dx = \dfrac{1}{a}\ln|\sec(ax+b)| + C$

(21) Solution for $\dfrac{dy}{dx} = f(x) \Rightarrow y = \int f(x)dx$ (22) Solution for $\dfrac{dy}{dx} = \dfrac{f(x)}{g(y)} \Rightarrow \int g(y)dy = \int f(x)dx$

(23) Solution for $\dfrac{dy}{dx} + Py = Q \Rightarrow y = \dfrac{1}{IF}\int IF\cdot Qdx$ where $IF = e^{\int Pdx}$

(24) Mean, $\bar{x} = \dfrac{\sum fx}{n}$ (25) Variance, $\sigma^2 = \dfrac{\sum fx^2}{n} - \left(\dfrac{\sum fx}{n}\right)^2$

(26) Standard deviation, $\sigma = \sqrt{\dfrac{\sum fx^2}{n} - \left(\dfrac{\sum fx}{n}\right)^2}$

QUESTION 1

[(5 + 5) + 8 + 7 = 25 Marks]

(a) Solve the following equations.

(i) $\log_4(3x-5)=2$ (ii) $e^x - 5e^{-x} = 4$

(b) Sketch the graph of the exponential function $y = 2^{-2x} - 4$ by clearly labelling the axes intercepts and the horizontal asymptote.

(c) The equation $Q(t) = 17e^{-0.025t}$ gives the mass Q in grams of the radioactive Potassium-42, that will remain from some initial quantity after t hours of radioactive decay. Roughly, how long will it take for 45% of the initial amount of potassium-42 to decay?

QUESTION 2

[(4 + 4) + (5 + 5) + 10 = 28 Marks]

(a) Find the derivatives of the following functions.

(i) $y = \sin(4-5x)$ (ii) $y = \ln\left(e^{3x-4x^2}\right)$

(b) Integrate the following functions.

(i) $\int 3x(5-3x^2)^4\,dx$ (ii) $\int_0^{\frac{\pi}{2}} 4x\cos(2x)\,dx$

(c) Find the area of the region enclosed by the curves $y = x^2 - 9$ and $y = 2x - 6$.
[Note that a sketch of the required region is necessary]

QUESTION 3

[(4 + 6) + (4 + 3 + 3) = 20 Marks]

(a) Solve the following first order differential equations

(i) $(x-3)\dfrac{dy}{dx} = y + 2.$ (ii) $\dfrac{dy}{dx} - 2y = e^{3x},\ y(0) = 5.$

(b) The world population in 1998 was approximately 5.9 billion and growing at a rate of about 1.33% per year.

(i) Form a first order differential equation (IVP) that represents the population phenomena.

(ii) Solve the equation in part (i).

(iii) Use your answer in part (ii) or otherwise to estimate the world population by year 2038.

QUESTION 4

[5 + 7 = 12 Marks]

Solve the following second order differential equations.

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0.$

(b) $\frac{d^2y}{dx^2} + 9y = 0, y(0) = 1, y'(0) = -2.$

QUESTION 5

[7 + 2 + (3 + 3) = 15 Marks]

The number of components processed in one hour on a new machine was recorded on 40 occasions as shown below.

66	87	79	74	84	72	81	78	68	74
80	71	91	62	77	86	87	72	80	77
76	83	75	71	83	67	94	64	82	78
77	67	76	82	78	88	66	79	74	64

(a) Construct a grouped frequency table with 7 classes.

(b) Using your answer in (a) above answer the following:
- (i) State the model class.
- (ii) Calculate the mean.
- (iii) Calculate the standard deviation.

END OF EXAM