PAPUA NEW GUINEA UNVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATIONS - 2021

FIRST YEAR APPLIED SCIENCE MATHEMATICS

MA125 – MATHEMATICS 2 AS (A)

TIME ALLOWED: 3 HOURS

INSTRUCTIONS FOR CANDIDATES

1. You have 10 minutes to read this paper. You are not to write during this time.

2. This examination consists of two sections:

    Part A – 15 Multiple Choice Questions worth 2 marks each to give a total of 30 marks.

    Part B – 7 Long Answer Questions worth 10 marks each to give a total of 70 marks.

3. Write ALL answers in the answer booklet provided.

4. For **Multiple Choice**, organise first page (page 1) of your answer booklet by numbering 1 – 15 and write the correct letter of your answer next to question number.

5. There are fifteen (15) multiple choice questions and you are to write the letter A, B, C or D of the correct answer onto the first page (page 1) of the answer booklet provided.

6. There are seven (7) long answer questions. Start long answer questions on page 2 of the answer booklet.

7. Start each question of Part B (Long Answer Questions) on a new page and clearly write its question number at the top of the page.

8. Show all necessary working out in the booklet provided.

9. Scientific and business calculators are allowed.

10. Write your name and ID number clearly on the examination answer booklets and sign off.

11. **Mobile phones** must be **switched off** during the examination period.

Marking Scheme

Marks are as indicated at the beginning of each question.

Total Mark is **100**

PART A – MULTIPLE CHOICE:

[2 x 15 = 30 marks]

Write the correct letter **A, B, C** or **D** next to question number in the first page (page 1) of your examination answer booklet.

Question 1   The general equation of an exponential function is $y = Aa^{kx} + b$. The condition for a decay function is when:

A $a > 0, k > 0$ B $a > 1, k > 1$ C $a > 1, k > 0$ D $a > 1, k < 0$

Question 2   You deposit K200 into a bank account. Every year that account increases exponentially by 10%. What is the equation of the investment?

A $y = 210(1.10)^x$ B $y = 200(1.10)^x$ C $y = 210(0.10)^x$ D $y = 200(0.10)^x$

Question 3   If $\frac{dy}{dx} = 6x^5 - 5x^4$; what will be the expression for $\frac{d^2y}{dx^2}$?

A $x^6 - x^5$ B $x^4 - x^3$ C $30x^4 - 20x^3$ D $30x^6 - 20x^5$

Question 4   The derivative of $f(x) = e^{2x} + \sin x$ is:

A $2e^{2x} + \cos x$ B $2e^{2x} - \cos x$ C $e^{2x} + \cos x$ D $e^{2x} - \cos x$

Question 5 $\int (6x^5 - 3x^2)dx =$

A $30x^6 - 6x^3 + c$ B $x^6 - x^3 + c$ C $30x^4 - 6x + c$ D $x^5 - x^3 + c$

Question 6 $\int_1^3 (2x)dx =$

A   9   B   8   C   7   D   6

Question 7   What is the total sample space if you toss a six-face die 3 times?

A   18   B   36   C   72   D   216

Question 8   A and B are independent events. If $P(A \cap B) = \dfrac{2}{9}$ and $P(B) = \dfrac{1}{3}$; then $P(A)$ is:

A $\dfrac{1}{3}$ **B** $\dfrac{2}{3}$ **C** $\dfrac{1}{27}$ **D** $\dfrac{2}{27}$

Question 9   The value of $\binom{8}{4}$ is

A 60   **B** 65   **C** 70   **D** 75

Question 10   The average of a certain test is 35 and the standard deviation is 5.

What would be Paul's z–score if he score is 50 in that test?

A 2   **B** 3   **C** 4   **D** 5

Question 11   How many terms will be in the expansion of $(5x + 2)(7 - 3x)(2 - x)(2x - 3)$

A 4   **B** 8   **C** 12   **D** 16

Question 12   The test used to check if events A and B are independent is:

A $P(A \cup B) = P(A) \times P(B)$ **B** $P(A \cap B) = P(A) \times P(B)$

C $P(A \cup B) = P(A) + P(B)$ **D** $P(A \cap B = P(A) + P(B)$

Question 13   How many students sat for a test which has a mean (μ) of 30 and the sum of all the score to be 1560?

A 52   **B** 520   **C** 30   **D** 300

Question 14   The 2nd derivative of $y = 3x^2$ is:

A 6x   **B** 6   **C** x^3 **D** x^2

Question 15   The indefinite integral of $\int 2\,dx$ is:

A $0 + c$ **B** $2 + c$ **C** $2x + c$ **D** $2x^2 + c$

PART B. $[10 + 10 + 10 + 10 + 10 + 10 + 10 = 70 \text{ marks}]$

Write question number on a new page in the answer booklet and show all necessary working out.

Question 16 $[2 + 2 + 2 + 2 + 2 = 10 \text{ marks}]$

At the beginning of a population study, the population of Morobe Province was 300,000. Two years later, the population was 320,000. Assume the population grows exponentially according to the function $P(x) = P_o e^{kt}$ where P_o is the initial population, P is the Population at a given time (t), k is the growth rate and t is time in years

a) What would be the growth rate (to 3 decimal places) of the population using the information given above?

b) Find the growth model by inserting the values of P_o and k into the function $P(x) = P_o e^{kt}$.

c) What would be the population (to nearest whole number) of Morobe 6 years after the start of the study?

d) How long (to one decimal place) after the start of study will the population be twice as much as when the study began?

e) Sketch the function P(x) with population along the y-axis and time, t, along the x-axis.

Question 17 $[2 + 4 + 4 = 10 \text{ marks}]$

The equation of a curve is $y = e^{2x} - \sin x$.

(a) Find the first derivative of the curve given above.

(b) Find the equation of the tangent to the curve at x = 0 and write the answer in the standard form.

(c) Find the equation of the normal to the curve at x = 0 and write the answer in the standard form.

Question 18 $[2 + 4 + 2 + 2 = 10 \text{ marks}]$

If the graph of $y = 3x^2 - 2x^3$ is given, then:

(a) find the y – intercept of the graph.

(b) find the stationary points (or turning points) of the graph.

(c) determine the nature of each turning point with working out.

(d) sketch the graph of $y = 3x^2 - 2x^3$ clearly showing the y-intercept and the turning points.

Question 19 $[5 + 5 = 10 \text{ marks}]$

If $f(x) = 2x^2$ and $g(x) = 2x + 4$, then:

(a) find the points of intersection between f(x) and g(x) and

(b) find the area between the two curves where $g(x) \geq f(x)$.

Question 20   [5 + 5 = 10 marks]

A and B are two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{x}{4}$ and $P(A \cup B) = \frac{3x}{4}$.

(a) Find the value of x which makes events A and B <u>mutually exclusive</u> to each other and hence write down the values of P(B) and P(A ∪ B).

(b) Find the value of x which makes events A and B <u>independent</u> to each other and hence write down the values of P(B) and P(A ∪ B).

Question 21   [5 + 5 = 10 marks]

Given that $\frac{d^2y}{dx^2} = -10 \sin 2x + 8 \cos 2x$, find the:

(a) equation of the first derivative when $x = \pi$ and $\frac{dy}{dx} = 20$, and

(b) primitive (or original) function, y = f(x), when x = 0 and y = 6

Question 22   [3 + 4 + 2 + 1 = 10 marks]

Below are Mathematics test score for 25 students:

| 20 | 15 | 13 | 17 | 14 | 15 | 16 | 20 | 15 | 12 | 18 | 19 | 15 |
| 14 | 20 | 13 | 15 | 18 | 14 | 15 | 20 | 19 | 15 | 13 | 15 | |

(a) Arrange the above data using a frequency table with headings as given below.

Score (x)	Frequency (f)	Frequency x score (fx)	Deviation ($|x - \mu|$)	Squared deviation ($|x - \mu|^2$)

(b) Calculate or find the range, mode, median and the mean from the frequency table.

(c) Calculate the variance, standard deviation from the frequency table.

(d) Calculate the z-score for a student who scored 18 in the Mathematics test.

......................**End of Semester 2 Examination** ....

Function	Derivative (dy/dx)	Integral
	$\dfrac{dy}{dx} = f'(x) = g'(x)$	$\int f(x)dx + c = F(x) + c$
$f(x) = ax^n$	$f'(x) = nax^{n-1}$	$F(x) = \dfrac{ax^{n+1}}{n+1} + c$
$f(x) = a \sin bx$	$f'(x) = ab \cos bx$	$F(x) = -\dfrac{a}{b} \cos bx + c$
$f(x) = a \cos bx$	$f'(x) = -ab \sin bx$	$F(x) = \dfrac{a}{b} \sin bx + c$
$f(x) = e^{bx}$	$f'(x) = be^{bx}$	$F(x) = \dfrac{1}{b} e^{bx} + c$

Probability Rules

Addition Law: $\quad\quad\quad\quad$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Law: $\quad\quad$ $P(A \cap B) = P(A) \times P(B)$

	Descriptive Statistics	Discrete Random Variable
Mean (μ)	$\dfrac{\sum fx}{\sum f}$	$\sum xP(x)$
Variance (σ²)	$\dfrac{\sum(x-\mu)^2}{\sum f}$	$\sum(x-\mu)^2 P(x)$
Standard Deviation (σ)	$\sqrt{\dfrac{\sum(x-\mu)^2}{\sum f}}$	$\sqrt{\sum(x-\mu)^2 P(x)}$
Notes:	$x - score$ $f - frequency$	$x - random\ variable$ $P(x) - probability$

Binomial Theorem: $\sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$ $where\ 0 \leq r \leq n$ $\quad$ $and\ also\ \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Binomial Probability Distribution: $\quad P(x=r) = \binom{n}{r}p^r q^{n-r}\ where\ \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Combination Formula: $\quad\quad\quad \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Z – Score $\quad = \quad \dfrac{x - \mu}{\sigma}$