THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

FIRST SEMESTER EXAMINATIONS – 2023

SECOND YEAR BACHELOR OF TECHNOLOGY IN SURVEYING & BACHELOR OF GEOGRAPHIC INFORMATION SCIENCE

MA 215 – MATHEMATICS 2 SV

TIME ALLOWED: 3 HOURS

INSTRUCTIONS FOR CANDIDATES

1. You have 10 minutes to read through this paper. You must **NOT** begin writing during this time.
2. There are six (6) questions. Answer **ALL** questions.
3. Write all answers in the answer booklet(s) provided.
4. All workings should be shown clearly in the answer booklet(s).
5. Start each question on a new page and clearly write its question number at the top of the page.
6. Calculators are allowed in the examination room.
7. Mobile phones **must** be switched off during the examination period.
8. Make sure that your **name, surname** and **ID number** are clearly written on the front of the examination answer booklet(s).
9. Check to see that a formula sheet is attached.

MARKING SCHEME

Questions carry marks as indicated. Total marks is **100.**

Question 1: $[5 + 5 = 10 \text{ marks}]$

(a) Given $f(x, y) = y^3 e^{-5x}$, calculate $f_{xy}(0,1)$.

(b) Given $y = e^{-x^2}$, find the expression for $\frac{d^2y}{dx^2}$.

Question 2: $[4 + 5 + 4 + 3 = 16 \text{ marks}]$

For the curve function $x^3 + y^3 = 9$, answer the following questions:

(i) Find $\frac{dy}{dx}$ at the point $(1,2)$.

(ii) Find $\frac{d^2y}{dx^2}$ at the point $(1,2)$.

(iii) Calculate the curvature at the point $(1,2)$, giving your answer correct to 4 decimal places.

(iv) Calculate the radius of curvature at the point $(1,2)$, giving your answer correct to 4 decimal places.

Question 3: $[5 + 8 + 5 = 18 \text{ marks}]$

Given the function $y = \frac{x}{x^2+1}$;

(i) find $\frac{dy}{dx}$ at $x = 0$.

(ii) find the stationary points of function and determine its nature.

(iii) find the definite integral of the function from $x = 2$ to $x = 3$. (give answer in simplified ln)

Question 4: $[4 + 6 + 6 = 16 \text{ marks}]$

A curve C is defined by these parametric equations:

$x = \sin t$ and $y = \cos(2t)$, for $0 \leq t \leq \pi$ where t is a parameter.

(i) Find the expression for $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(ii) Find the gradient of the tangent at the point P on the curve where $t = \frac{\pi}{6}$.

(iii) Find the equation of the normal to the curve at the point P on the curve where $t = \frac{\pi}{6}$.

Question 5: $[6 + 6 = 12 \text{ marks}]$

Given $y = x \ln x$;

(i) calculate its indefinite integral using formula: $\boldsymbol{I = uv - \int u'v\, dx}$.

(ii) calculate its definite integral from $x = 1$ to $x = 2$, giving your answer correct to 4 decimal places.

Question 6: $[(9 + 5 + 6) + 8 = 28 \text{ marks}]$

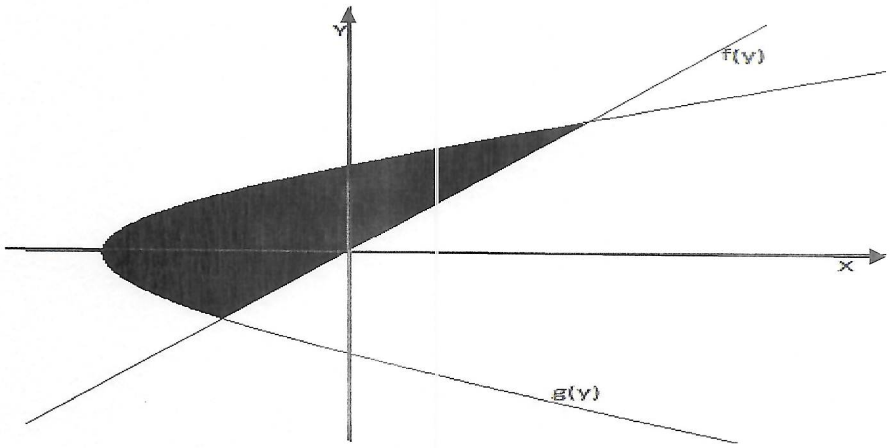
(a) The linear function $f(x) = x - 3$ and quadratic function $g(x) = x^2 - 3x$ intersects at two points.

(i) Sketch the graphs of both functions on the same set of axis clearly showing these features: $x \,\&\, y$ *intercepts, stationary point and points of intersection.*

(ii) Find the **exact** area of the region enclosed by the two graphs.

(iii) Find the volume of the solid formed by revolving the region between $f(x)$ and $g(x)$ about the x-axis.

(b) Find the area of the shaded region enclosed by the straight line $f(y) = y$ and the curve $g(y) = y^2 - 2$, as shown below:



END OF EXAMINATION

Formula Sheet

Name of Rule	Formula
Area under the curve	$A = \int_a^b f(x)dx$
Area between 2 curves	$A = \int_a^b [f(x) - g(x)]\,dx$
Volume of Solid	
(i) Rotation about x-axis (ii) Rotation about y-axis (iii) Regions bounded by two functions	(i) $V = \pi \int_a^b y^2 dy$ (ii) $V = \pi \int_a^b x^2 dy$ (iii) $V = \pi \int_a^b ([f(x)]^2 - [g(x)]^2)\,dx$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Function	**Integral**
$\cos x$	$\sin x + c$
$\sin x$	$-\cos x + c$
e^{ax+b}	$\frac{1}{a}e^{ax+b} + c$
$cos\ (ax+b)$	$\frac{1}{a}\sin(ax+b) + c$
$\frac{1}{x}$	$\ln x$+c
$\frac{1}{ax+b}$	$\frac{1}{a}\ln(ax+b) + c$
Function	**Derivative**
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$cos[f(x)]$	$-sin[f(x)].f'(x)$
$sin[f(x)]$	$cos[f(x)].f'(x)$
Product Rule	$u'v + uv'$
Quotient Rule	$\frac{u'v - uv'}{v^2}$
Curvature	$k = \frac{\left\lvert\frac{d^2y}{dx^2}\right\rvert}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$
Radius of Curvature	$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left\lvert\frac{d^2y}{dx^2}\right\rvert}$
Two perpendicular Lines	$M_1M_2 = -1$
Equation of line	$y - y_1 = m(x - x_1)$