PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

MATHEMATICS AND COMPUTER SCIENCE DEPARTMENT

SECOND SEMESTER EXAMINATIONS - 2021

SECOND YEAR PROPERTY STUDIES MATHEMATICS

MA226 – QUANTITATIVE METHODS II

TIME ALLOWED: 3 HOURS

INSTRUCTIONS FOR CANDIDATES

1. You have 10 minutes to read this paper. You are not to write during this time.
2. This examination is divided into two sections:

   Part A – 15 Multiple Choice Questions worth 2 marks each = 30 marks.

   Part B – 7 Long Answer Questions worth 10 marks each = 70 marks.
3. Write ALL answers in the answer booklet provided.
4. For Multiple Choice, organise first page of your answer booklet by numbering 1 – 15 and write the correct letter of your answer next to question number.
5. There are fifteen (15) multiple choice questions and you are to write the letter A, B, C or D of the correct answer onto the first page of the answer booklet provided.
6. There are seven (7) long answer questions. Start longer questions on page 2 of the answer booklet.
7. Start each question on a new page and clearly write its question number at the top of the page.
8. Show all necessary working out in the booklet provided.
9. Write your name and ID number clearly on the examination answer booklets.
10. **Mobile phones** must be **switched off** during the examination period.

MARKING SCHEME

Total mark is out of 100.

PART A – MULTIPLE CHOICE: Write the correct letter **A, B, C** or **D** next to question number in your answer booklet.

Question 1. Which of the fraction below is a valid probability reading?

A -8/9 **B** 9/8 **C** -9/8 **D** 8/9

Question 2. How many possible outcomes are there for the experiment **"Rolling of a fair die 4 times"**?

A 1296 **B** 216 **C** 36 **D** 6

Question 3. Suppose a card is drawn from a deck of 52 cards. What is the probability of drawing a 10 from a black suit?

A 1/52 **B** 4/13 **C** 1/26 **D** 1/13

Question 4. If you toss a coin three times, what is the probability of observing at least two heads?

A 1/8 **B** 2/8 **C** 3/8 **D** 4/8

Question 5. A sample of 4 carrots is chosen from a bag of 20 carrots. The number of rotten carrots in the sample is counted. What is the sample space for this experiment?

A S={0,1,2,…,20} **B** S={0,1,2,3,4} **C** S={1,2,3,4} **D** S={1,2,3,…,20}

Question 6. If A = {1,2,3,4}, and B = {2,4,6,8}; what is ***A union B***?

A {1,2,2,3,4,4,6,8} **B** {1,2,3,4,6,8} **C** {2,4} **D** {1,3,6,8}

Question 7. Which of the following represents a graphical presentation of a categorical data?

A Pie Chart **B** Scatter Plot **C** Histogram **D** None of the given choices

Question 8. A histogram with two peaks is a …….. histogram.

A unimodal **B** left skewed **C** bimodal **D** right skewed

Question 9. An example of a numerical variable is a student's

A gender **B** age **C** eye color **D** year level

Question 10. If X = {9,2,4,1,6,5,7,3}, what is the range for X?

A 4.5 **B** 6 **C** 7 **D** 8

Question 11. What measure of central tendency splits a dataset into two equal groups?

A mean **B** range **C** median **D** mode

Question 12. What do you think will influence the calculation of a mean?

A Order of data **B** Outliers **C** Variance **D** Individual scores

Question 13. If there were a very strong, positive correlation between two variables, the correlation coefficient (r) would be:

A -0.25 **B** +1 **C** +0.25 **D** -1

Question 14. A test has a mean of 20 with a standard deviation of 2. Suppose a student scored 18, what would be the student's z-score?

A -2 **B** 2 **C** -1 **D** 1

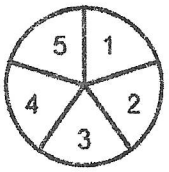
Question 15. If the random variable X follows a Binomial distribution with n = 10 and $\pi = 0.5$. What is the expected value of X?

A 10 **B** 5 **C** 0.5 **D** 0.1

PART B. Write question number on a new page in the answer booklet and show all necessary working out.

Question 16 **[2+2+2+2+2 = 10 marks]**

Consider the "wheel of fortune" illustrated on the right. The wheel is spun about an axis at the centre of the wheel, and one of the numbered sectors lands under the arrowhead. We are interested in the number under the arrowhead.



(a) List all the outcomes of this experiment.

(b) What is the probability of each outcome?

(c) Find the probability that the number under the arrowhead is less than or equal to 4.

(d) Find the probability that the number under the arrowhead is even.

(e) Find the probability that the number under the arrowhead is divisible by 2.

Question 17 **[2+2+3+3 = 10 marks]**

Urn X has 3 red and 2 blue marbles; Urn Y has 5 red and 10 blue marbles. The experiment is to randomly choose an urn and to randomly draw a marble from the chosen urn.

(a) Draw a tree diagram for this experiment.

(b) Label the tree diagram with probabilities.

(c) For a randomly selected marble, what is P(red|X)

(d) For a randomly selected marble, what is P(blue|Y)

Question 18 **[2+2+2+4 = 10 marks]**

Suppose a computer shop sells one of its products according to the following probability distribution

x	0	1	2	3
P(x)	2/8	3/8	2/8	1/8

Where its profit follows the function $h(X) = 500X - 200$.

(a) Calculate the expected value for X.

(b) Calculate the variance for X

(c) Calculate the standard deviation for X

(d) Calculate the expected profit that the company will make from selling the product.

Question 19 **[2+2+2+2+2 = 10 marks]**

A child psychologist is interested in the number of times a newborn baby's crying wakes its mother after midnight. For a random sample of 20 mothers, it was observed that a baby may wake its mother 5, 4, 3, 2, 1, or 0 times with 1/20, 4/20, 8/20, 4/20, 2/20, 1/20 probabilities respectively.

(a) Create a probability histogram for the number of times a newborn baby's crying wakes its mother after midnight.

(b) Create a cumulative probability distribution table for the number of times a newborn baby's crying wakes its mother after midnight.

(c) What is the probability that a child's crying wakes its mother more than 3 times?

(d) What is the probability that a child's crying wakes its mother between 0 and 2 times?

(e) What is the probability that a child's crying wakes its mother at least once?

Question 20 **[5+2+2+1 = 10 marks]**

Suppose that it is known that 40% of voters support the Conservative party. We take a random sample of 4 voters. Let the random variable X represent the number in the sample who support the Conservative party.

(a) Explain why the distribution of X is binomial.

(b) Complete the table below for the probability distribution of Y.

[Use $p(x) = \frac{n!}{(n-x)!x!} \times \pi^x \times (1-\pi)^{n-x}$]

x	0	1	2	3	4
P(X = x)					

(c) Find the mean of X.

(d) Find the variance of X.

(e) Find the standard deviation of X.

Question 21 **[2+5+3 = 10 marks]**

A radioactive source that follows a Poisson distribution emits 5 particles on average during a ten-second period.

(a) Calculate the probability that it emits 3 particles during the 10-second period.

(b) Calculate the probability that it emits at most 2 particles during the 10-second period.

(c) During a 20-second period, what is the probability that 6 particles are emitted?

Question 22 **[5+5 = 10 marks]**

Most tertiary institutions require applicants for admission to sit for the Special Tertiary Admissions Test (STAT-P). Scores on the STAT-P test are roughly normally distributed with a mean of 527 and a standard deviation of 112.

(a) What is the probability of an individual scoring 515 or more on the STAT-P test?

(b) What is the probability of an individual scoring between 500 and 510 on the STAT-P test?

.....End of Semester 2 Examination .....

	Descriptive Statistics	Discrete Random Variable	Binomial Experiment	Poisson Experiment
Mean (μ)	$\frac{\sum x}{n}$	$X: \sum xP(x)$ $h(X): \sum h(x)P(x)$	$n\pi$	λ
Variance (σ^2)	$\frac{\sum(x-\mu)^2}{n}$	$(x-\mu)^2P(x)$ or $E(X^2)-[E(X)]^2$	$n\pi(1-\pi)$	λ
Standard Deviation (σ)	$\sqrt{\frac{\sum(x-\mu)^2}{n}}$	$\sqrt{(x-\mu)^2P(x)}$ or $\sqrt{E(X^2)-[E(X)]^2}$	$\sqrt{n\pi(1-\pi)}$	$\sqrt{\lambda}$
Notes:	*x – score* *n – total number of scores*	*X – random variable* *P(X) – probability of X*	*n – fixed trials* *π: probability success* *1 – π: probability of failure*	*λ: success rate*

Binomial Probability Distribution: $p(x) = \frac{n!}{(n-x)!x!} \times \pi^x \times (1-\pi)^{n-x}$

where x = 0, 1, .., n and $0 \leq \pi \leq 1$

Poisson Probability Distribution: $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Where λ *is the mean number of occurences*

and X is the number of occurences

Z-score: $z = \frac{x-\mu}{\sigma}$

Probability

P(E)	$P(E) = \frac{n(E)}{n(S)}$
Exclusive Events (A or B)	$P(E) = P(A) + P(B)$
Independent Events (A and B)	$P(E) = P(A) * P(B)$
Conditional Probability	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$

Probabilities Associated with the Standard Normal Distribution
[ie, Areas under the Standard Normal Curve]

To read the table: use the rows to find the first digit, and the columns to find the second digit of a Z-score. To find 0.69, first look down the rows to find 0.6 and then across the columns to 0.09 and 0.7549 will be the result - the probability that the z-score is less than 0.69.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990