The Papua New Guinea University of Technology

Department of Mechanical Engineering

ME 212: Numerical Methods

Second Year First Semester Examination – 2023

Thursday, June 01, 2023 – 12:50 pm

Time Allowed: Two (2) Hours

Instructions

(1) You have **10** minutes to read the paper. **Do not** write anything during this time.

(2) There are **four (4)** questions. Answer **each** question.

(3) All questions must be answered in the booklet provided. No other written material will be accepted.

(4) Calculators are permitted in the examination room. Notes and books are **not** allowed. Any student found in cheating will be **disqualified**.

(5) Write your name clearly on the front page using block letters.

(6) All questions carry equal marks.

Q1. **(25 Marks)**

Let $a = 47.13$, $b = 4.185$ and $c = -0.645$. Use three digit arithmetic to compute $\frac{a + b * c}{b + c}$.
Identify the rounding error at each stage of the calculation and the total effect of rounding error.

Q2.

(a) Let a be a fixed positive real number. Define the sequence x_n by **(25 Marks)**

$$x_0 = 1 \qquad x_n = \frac{1}{2}\left(x_{n-1} + \frac{a}{x_{n-1}}\right),\ n = 1,2, \dots \dots.$$

Show that the sequence x_n is the same sequence that arises by using Newton's method on the function $f(x) = x^2 - a$ with $p_0 = 1$.

(b) Determine the fixed points of the function $g(x) = \frac{x^2 - 3}{2}$ and show them graphically.

Q3. **(25 Marks)**

(a) The following form of the polynomial is called Vandermonde polynomial of degree n :

$$P_n(x) = a_0 + a_1 x + \dots\dots\dots + a_n x^n .$$

Determine this polynomial with $n = 2$, which takes the values of f from the following table, to approximate f :

i	x_i	$f(x_i)$
0	0	0
1	1	1
2	2	0

(b) If $f(x) = \frac{\sin \pi}{2} x$ in part (a) then compute the maximum error in $[0,2]$.

Q4. **(25 Marks)**

(a) Derive two-point approximation to $f'(x_0)$ and the approximation error using the following formula for $n = 1$:

$$f'(x_j) = \sum_{k=0}^{n} f(x_k) L'_{n,k}(x_j) + \frac{f^{(n+1)}(\xi(x_j))}{(n+1)!} \prod_{\substack{k=0 \\ k \neq j}}^{n} (x_j - x_k)$$

(b) Derive the Euler's method and its truncation error using Taylor's theorem.

(c) Given the initial-value problem $y' = -y + t + 1$, $0 \leq t \leq 0.4,$ $y(0) = 1$,

with exact solution $y(t) = e^{-t} + t$. Using the Euler's method with $h = 0.2$, approximate $y(t_i)$ and calculate the actual error $|y(t_i) - w_i|$ at $t_i = 0.2$.