PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY (PNGUOT)
MECHANICAL ENGINEERING

ME212: Numerical Methods
First Semester Examination, 2022
Second Year Mechanical Engineering
Thursday, June 2nd, 2022- 12:50 P.M
Location: M114/M115

Time Allowed: 2 Hrs

Instructions:

1. *You have 10 minutes to read the paper. Do not write anything during this time.*
2. *Write your **name** clearly on the front page using **Capital letters**.*
3. *There are **total five (5) questions**. **Answer any four (4) questions**.*
4. *All questions carry equal marks.*
5. *All questions must be answered only in the booklet provided.*
6. *Calculators are permitted in the examination room.*
7. *Any student found cheating will be disqualified.*

Question 1: (25 Marks)

PART-A **(5 Marks)**

A. Define types of errors that occur in numerical methods?

PART-B **(20 Marks)**

B. Given the following equation:

$$x^4 + x - 10 = 0$$

(a) Use Newton-Raphson method to find the root correct to three decimal places with initial guess as$x_0 = 2$.
(b) Determine the approximate relative error after each iteration.

Question 2: (25 Marks)

A. Solve the system of linear equations using Gauss-elimination method with pivoting **[15 Marks]**

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

(b) Use Matrix Inversion method to solve the above system of linear equations **[10 Marks]**

Question 3: (25 Marks)

A. Use the given data to:

x	1	2	3	4	5
y	1	1.5	2	3	4

(a) Use Least-squares regression to fit a straight line. Find y at x=4.5.
(b) Interpolate f (4.5) using Lagrange Polynomials of order 2.
(c) Compare (a) and (b)

Question 4: (25 Marks)

A. The following data was collected for the velocity versus time for a rocket:

t, s	0	25	50	75	100	125
V, km/s	0	32	58	78	92	100

(a) Use numerical differentiation to estimate the rocket's acceleration at t=25 sec, t=100 sec respectively.
(b) Use Single application Simpsons 1/3 rule, numerical integration to find the Distance travelled by rocket from t=0 sec to t=50 sec.

Question 5: (25 Marks)

A. Compute y (0.4), where y (1) =0.

$$\frac{dy}{dt} = 1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2 \quad 1 \le t \le 3$$

(a) Euler's method with h=0.2
(b) Fourth-order RK method with h=0.2.

Helpful Hints:

1. $P_n(x) = \sum_{i=0}^{n} f_i l_i(x)$ ... (v)
   $l_i(x) = \prod_{j=0, j\neq i}^{n} \left(\frac{x-x_j}{x_i-x_j}\right)$ ... (vi)

2. $y_{i+1} = y_i + f(x_i, y_i).h$

3.

$m_1 = f(x_i, y_i)$

$m_2 = f\left(x_i + \frac{h}{2},\ y_i + \frac{m_1 h}{2}\right)$

$m_3 = f\left(x_i + \frac{h}{2},\ y_i + \frac{m_2 h}{2}\right)$

$m_4 = f(x_i + h,\ y_i + m_3 h)$

Then, the general form of extrapolation equation (i) will be:

$y_{i+1} = y_i + \frac{h}{6}(m_1 + 2m_2 + 2m_3 + m_4)$ ... (ii)

4. $I \quad = \frac{h}{3}(f_0 + 4f_1 + f_2)$

5. $y = a + bx$

$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$ .... (iv) & $a = \frac{\sum y_i}{n} - b\frac{\sum x_i}{n}$

Good luck!!!